The Value of Adaption: Climate Change and Timberland Management

Christopher Guo and Christopher Costello *

December 7, 2012

Abstract: Adaptation to exogenous change occurs on both intensive and extensive margins. Whether and how one accounts for human adaptation directly affects estimates of the economic consequences of environmental change, estimates that are both critical in informing policy decisions and notoriously difficult to value. This paper introduces and applies an analytical framework for placing an economic value on adaptation. We explore the issue first in a stylized model that facilitates making concrete generalizations about the kinds of adaptations that generate high or low economic value. We then test the soundness of our insights by incorporating learning and adaptive decision-making into a structural dynamic forestry model where climate change is imposed exogenously and agents respond optimally. Using downscaled climate projections integrated with site- and species-specific timber productivity data, we estimate the economic value of adaptation to climate change within the California timber industry. We find on the intensive margin, changing the rotation intervals will yield a low value of adaptation, but on the extensive margin, replanting more suitable tree species can yield significant value.

Keywords: Adaptation; Economic Impact Analysis; Environmental Change; Climate Change; Forestry; Faustmann; Extensive Margin; Intensive Margin.

---

*Guo: Department of Economics, University of California Santa Barbara, 2120 North Hall, Santa Barbara CA 93106 *guo@econ.ucsb.edu* 805-617-0844; Costello: Bren School of Environmental Science and Management, University of California Santa Barbara, 4410 Bren Hall, Santa Barbara CA 93106 and NBER *costello@bren.ucsb.edu*. We gratefully acknowledge two anonymous referees, Charlie Kostad, C.Z. Qin, Martin Smith, Lee Hannah, and seminar participants at UC Davis for helpful suggestions. For developing GIS maps we thank Ming Ng and Isaac Pearlman. For financial support we acknowledge the California Energy Commission and the UCSB ConvEne IGERT.

1 Introduction

Extreme weather events, catastrophic fires, sea level rise, ocean acidification, species range shifts, floods, famine, and infestations are just a few of the hundreds of documented consequences of climatic change. For policy analysis, a fundamental contribution of economics is to estimate the economic impacts of environmental changes, which are then used as inputs into cost benefit calculations for policy design (Stern 2006; Nordhaus 1994; Mendelsohn et al. 2000; Tol 1995; Frankhauser 1995). Regardless of the cause or specific context, many environmental changes share underlying patterns. First, environmental change is dynamic; the external stimulus of environmental change, whether it be a land management practice or pollution, has an intertemporal path. The stimulus then causes different outcomes within the ecological systems over time due to lags and adjustments. Second, the market response to environmental change is dynamic, as individual economic agents, firms, and governments make adaptive decisions. Due to these two underlying characteristics, the economic effects of climate change are extremely difficult to quantify, as seen in the wide range of impact estimates (Tol 2009).

That humans will adapt to exogenous change seems uncontroversial; predicting human behavior is a cornerstone of microeconomic theory. Clearly, failing to properly account for adaptation is a concern because impact estimates depend fundamentally on whether and how adaptation is accounted for. Damages will be overstated (and benefits understated) if adaptation is ignored. While it seems intuitive that impact assessments may hinge critically on if and how adaptation is incorporated, this expectation has not been formally evaluated. This paper introduces an analytical framework for measuring the *economic value of adaptation* given different types of dynamic adjustment. We define adaptation as a behavioral change or economic investment made to avoid or limit economic damages (if the environmental change is deleterious) or to enhance welfare (if the environmental change is welfare-enhancing). A central focus of this analysis is to distinguish between adaptation on the intensive margin (where behavioral changes adjust to a continuous choice variable) and extensive margin (where discrete changes are made). We use the model to derive several theoretical insights about the conditions under which adaptation will have high or low value. We apply the framework to a more sophisticated structural dynamic decision problem within forestry in the presence of exogenous environmental change and stochastic timber prices.

We find results from the simple model are highly consistent with those from the fully dynamic decision problem. In combination, this analysis provides empirical estimates of the practical value of adaptation, general theoretical insights regarding adaptive behavior, and a new perspective on assessing impacts of environmental change.

The forestry sector provides an ideal case to test our theory. 150 years ago, the German forester Martin Faustmann derived a formula to calculate the present value of income streams over an infinite cycle of forest rotations. Modern work concerning the optimal rotation has increasingly focused on risk and uncertainty analysis, given size-dependent stochastic growth (Clarke and Reed 1989; Reed and Clarke 1990) and stochastic timber prices, either using a reservation price policy, assuming prices can be modeled as independent draws from a known distribution (Brazee and Mendelsohn 1988; Lohmander 1988; Haight and Smith 1991), or taking a real options approach to the valuation of forest resources (Brennan and Schwartz 1985; Clarke and Reed 1989; Morck et al. 1989; Thomson 1992; Provencher 1995; Plantinga 1998; Insley 2002; Alvarez and Koskela 2007).

Climate change further introduces interesting dynamics into this story by changing site characteristics over time. Rainfall, temperature, or humidity are expected to shift in predicable ways over time and space, thus changing the suitability of any given site for growing any given species. The forester now faces a substantially more complex tradeoff because he must optimize the management of the tree stand with time-dependent growth and economic parameters in addition to uncertainty about future timber prices. Foresters are equipped with two fundamental adaptive responses to these kinds of changes. On the intensive margin, adaptation consists of adjusting the rotation period, the classical control variable in forestry. On the extensive margin, adaptation involves harvesting ones' tree stock and replanting a different species of tree altogether (Sedjo 2010). Sohngen and Mendelsohn (1998) use optimal control theory to calculate the optimal harvesting of timber species, while comprehensively accounting for species selection, dieback, and salvage harvests. They estimate welfare outcomes with a steady state response and with dynamic adjustments. We extend their work by focusing specifically on adaptive decisions, being careful to distinguish between the intensive and extensive margins. We develop a model of a forester's choice set at any point in time in the form of a Bellman equation. Pioneered by Provencher (1995) and Plantinga (1998), the application of dynamic programming to the forestry sector is useful for determining option value and is well-suited to modeling a full range of adaptation options on both margins. This paper contributes to the literature first, by integrating the Bellman approach to the Faustmann problem with climate change dynamics, second, by comparing theoretically and in an example the value of different types of dynamic adaptations, and third, by incorporating Bayesian learning about future timber price paths into a decision-making model under market uncertainty.

1.1 Literature on Adaptation in the Context of Climate Change

While adaptation is incorporated in ad hoc examples, there is little consistency in the literature on the definition of adaptation or on how to properly treat it. Economists either refer to adaptation as the learning process by which an agent's demand function is determined (Lucas 1986; DeBondt and Thaler 1994; Holland and Miller 1991; Bullard and Duffy 1998), as a hedonic treadmill where expectations rise in tandem with changes in fortune (Mochon et al. 2008; Loewenstein and Ubel 2008), or as a natural selection mechanism used to theoretically explain the size dynamics and distribution of firms (Rossi-Hansberg and Wright 2007; Costinot et al. 2009). In the environmental literature, adaptation is often treated indirectly as a possible decision to invest in a new energy or agricultural technology (Jaffe and Stavins 1994; Howarth et al. 2000; Griliches 1957; Besley and Case 1993; Foster and Rosenzweig 1995), or it is described as an adjustment cost attributed to the time it takes for inputs to change in the short run (Kelly et al. 2005). Within the specific context of climate change, adaptation is treated differently within the various impact models.

Few studies have considered dynamic responses of economic agents to steadily increasing concentrations of greenhouse gases. The economics literature on climate change mostly consists of impact assessments and traditionally follows a top-down approach where global greenhouse emission concentrations are fed into climate models, which deliver estimates of future climate conditions (mainly temperature and precipitation attributes) at a coarse spatial scale. Downscaling the global climate model to local climate conditions, researchers use this information to predict impacts. At this final stage, various degrees of adaptation might be introduced as part of an impact model.

The impacts literature treats adaptation in four classes: no adaptation, arbitrary adaptation, observed adaptation (analogues), and modeled adaptation. While much attention has been paid to the ecosystem dynamics of environmental change, many prominent economics papers implicitly assume either no adaptive response or perfect adaptation without investigating specifically which one was most appropriate (Easterling et al. 1993; Rosenzweig and Parry 1994; Adams 1989;

Fankhauser 1994; Frankhauser 1995; Nordhaus 1994; Tol 1995; Tol 2002). Later authors use observed adaptation to match temporal and spatial analogues. Spatial analogues compare cities A and B in the same period but different locations in order to predict how town A will change in a different time. This takes into account real adaptation as carried out by people in response to real climates (Mendelsohn et al. 1994; Deschenes and Greenstone 2007). Mansur et al. (2008) estimate a national energy model of fuel choice and consumption in order to determine the impact of climate change. Their cross-sectional data allows them to track long term adaptation, as households adjust not only immediate energy use but also capital stock. While the hedonic approach is able to determine the net effect of structural environmental change—i.e., the change in value from a change in temperature, it lacks an estimate of the value of adaptation, and thus provides no guidance about the conditions under which adaptation is economically important, which is the focus of this study. Because the net effect of environmental change implicitly includes adaptation, it is unclear how to separate the direct effect of the environment on status quo productivity from the value gained by substitution of inputs, introduction of different behavior, and other potential adaptations.

A complement to the hedonic approach is to model the dynamic process of adaptation. An influential paper by Sohngen and Mendelsohn (1998) estimated welfare effects with and without dynamic adjustments to climate change, and we further delve into the difference between these two welfare outcomes by exploring the effects of different types of dynamic adjustment. We define the value of adaptation as the difference of the value of economic activity when including a response and the value when ignoring a response:

$$\text{Value w/ adaptation} = \text{Value w/ out adaptation} \; + \; \text{Value of adaptation} \tag{1}$$

Determining the value of adaptation is particularly challenging because adaptation to climate is inevitably tied to a host of additional factors such as market pressures, environmental regulations, and insurance markets. Adaptation to climate is embedded in building construction, transportation systems, agriculture, leisure activities, and many other parts of daily life which are somehow structured to take into account prevailing climactic conditions. In the decision making process, many other factors exist and compete with climate change. For example, climate change adaptation in agriculture is embedded in the overall risk management strategies for a firm. Our theoretical model is highly stylized and abstracts away from this degree of contextual detail, but our application involves a structural dynamic decision model with endogenous adaptive decision-making. Climate change is imposed exogenously and agents respond optimally so that we can back out the value of adaptation. We are able to compare outcomes from decisions given environmental change and decisions in the absence of change, just like a controlled experiment, in order to explicitly isolate the value of adaptation.

In Section 2, we distinguish between adaptation on the intensive versus extensive margins. In Section 3, we show inherent differences between the two margins in a simple model to gain insight concerning the value of adaptation on each margin. In Section 4, we build back the dynamic complexity with a forestry application to climate change adaptation. We then incorporate learning and uncertainty into the application using a Bayesian stochastic dynamic programming approach to a classic forest rotation problem.

2 Two Margins of Adaptation

It seems a powerfully general rule that humans adapt to change both by adjusting the relative inputs using existing capital and by investing in new capital. We refer to the former approach as adapting on the intensive margin and the later as adapting on the extensive margin. A fundamental paper by Mendelsohn et al. (1994) first motivated extensive margins to climate change when exploring alternative crops and land use as agricultural responses to climate change; several subsequent papers have focused only on the intensive margin (Provencher 1995; Plantinga 1998; Insley 2002; Newman 2002). Many studies (both structural and spatial analogue approaches) describe increasingly sophisticated opportunities for adaptation along both margins, with the assumption that adding more adaptations will always affect economic impact estimates (Adams et al. 1990; Adams et al. 1998; Smith et al. 2000; Sedjo 2010). However, without disentangling the relative values of different types of adaptation, we lose the ability to determine when different types of adaptation are more advantageous. We find that the value of adaptation is fundamentally different on the intensive versus extensive margins, and under certain conditions, incorporating adaptation along either margin has no effect on impact estimates.

To illustrate the crucial distinction between the intensive and extensive margins, consider the adaptation strategies available to a motorist who experiences an increase in the price of gasoline. A well-documented response in the short run is to simply drive less (Gillingham 2012). This intensive margin adaptation does not require any structural shifts in capital, and may be achieved by car pooling, bundling car trips, or other means, with relatively minor economic consequences. Consider on the other hand adaptation on the extensive margin. Examples may include purchasing a more fuel efficient vehicle or switching the mode of transportation altogether to a train or bus. These structural shifts contain two features on which we will focus here. First they are all lumpy — only a small discrete set of choices is available. Second, they are acted upon only after the exogenous shock (here, the gas price increase) exceeds some threshold level.

Consider a second example of managing water resources in the face of climate-induced changes, which will require adapting to both heavier but less frequent precipitation and changes in water demand due to shifting population and land use. During the long droughts in between the floods, water supply management is paramount, and intensive adaptation could entail any change in behavior or policy tool that would adjust individual water consumption, such as pricing, rationing, or restrictions on nonessential or recreational water use. However optimal adaptation may also involve extensive adaptation such as the construction of new reservoirs, conveyance facilities, agricultural drainage management, stormwater reuse, or desalination plants (Hanak and Moreno 2012).

These examples are not cherry-picked. They illustrate the generality of the principle that when faced with structural exogenous change, economic agents may optimally respond (i.e. adapt) along both the intensive and extensive margins. Of course there are possibly small incremental (intensive) capital changes and large discrete (extensive) behavioral changes, but the only distinction we require is that some forms of adaptation are lumpy, and some are continuous in nature. Of course, the more options for discrete change, the more the extensive margin will approach the intensive (continuous) margin, e.g., if there were an infinite number of tree species with subtly-different genetic strains from which to choose. The remainder of this paper is devoted to calculating and illustrating the economic significance of intensive vs. extensive margin adaptation.

3 A Stylized Model

We begin by developing a stylized model to introduce the concept of using both continuous and discrete choice variables to model different types of adaptation. Stripping away the dynamic complexity of decision-making under changing conditions will allow us to glean general insights about the value of adaptation. Suppose an economic agent wishes to maximize a function f where x denotes a discrete choice variable, r denotes a continuous choice variable, and θ is an exogenous parameter. The value function over the parameter θ is given simply by:

$$\begin{aligned} V(\theta) &= \max_{x,r} f(x, r, \theta) \\ &= f(x^*(\theta), r^*(\theta), \theta) \end{aligned} \tag{2}$$

A key point of departure for this paper is recognizing that the adaptation choices made on the intensive and extensive margins must be accounted for differently in the optimization. In fact, a recurring theme from climate change adaptation is that the intensive margin adaptations allow for a continuous range of possible choices while extensive margin adaptations fall into discrete choice options. This simple model leads immediately to our first result.

Proposition 1 *The marginal value of adaptation on the intensive margin is 0.*

Formal proofs of all propositions are available in Appendix A. The proof relies on a simple application of the Envelope Theorem and takes advantage of the fact that $\frac{\partial f}{\partial r} = 0$ since the choice of the continuous variable r is optimal prior to the change in θ. Here intensive margin adaptation does occur (i.e. $\frac{\partial r}{\partial \theta} \neq 0$), but the value of adaptation on the intensive margin is zero. Essentially, because agents behaved optimally prior to the exogenous change, adaptation to a small change has no economic value. *Here ignoring adaptation has no effect on estimates of the economic impacts of the exogenous change.* Thus small structural shifts in the environment will be responded to by economic agents, but will not have appreciable economic consequences.

The classical treatment of the Envelope Theorem presented above applies only to continuous choices, $r(\theta)$. To accommodate the extensive margin, captured by the choice variable x, we will draw on recent literature that derives envelope theorems for use with discrete and arbitrary choice sets (Milgrom and Segal 2002; Sah and Zhao 1998). The value of adaptation on the extensive margin may behave differently due to the discrete nature of the choice variable and depending on whether the change in θ causes a threshold to be crossed.

Definition 1 *Let a "threshold crossing" be defined as an instance when a change in the parameter from θ to $\theta + \Delta\theta$ will lead to a nontrivial change in $x^*(\theta)$ such that $x^*(\theta) \neq x^*(\theta + \Delta\theta)$.*

Using this definition leads to our first extensive margin result:

Proposition 2 *The marginal value of adaptation on the extensive margin is 0 for any change in the parameter, given a threshold point is not crossed.*

The intuition behind Proposition 2 is that when no threshold is crossed, no change is made on the extensive margin, so there is no value of adaptation on the extensive margin. Under such circumstances, one can safely ignore adaptation. However, when a threshold is crossed, this result changes fundamentally, as is shown in the following proposition:

Proposition 3 *The marginal value of adaptation on the extensive margin can be arbitrarily large even for a small change in the parameter, provided that a threshold point is crossed.*

While formally proving Proposition 3 requires detailed analysis (see Appendix A), the basic intuition is straightforward. When a change in the parameter θ causes a threshold to be crossed, the discrete choice $x^*(\theta)$ that was optimal at θ becomes suboptimal at $\theta + \Delta\theta$. An adaptation on the extensive margin represents a switch from the status quo non-optimal discrete choice x_1 to a new optimal choice x_2. In fact, we find that the value of adaptation can be discretely and arbitrarily large even as $\Delta\theta$ approaches 0. The key insight is that that the marginal value of adaptation on the extensive margin is the difference in the slopes of $f(x_1, r^*, \theta)$ and $f(x_2, r^*, \theta)$ at the threshold crossing, and this does not converge to 0. The top panel in Figure 1 graphs $f(x_2, r^*, \theta)$ and $f(x_1, r^*, \theta)$ against the parameter θ, with the threshold at the dashed line. The bottom panel graphs the difference in value, also against θ. The slope of this line is the marginal value of adaptation on the extensive margin, which is clearly positive. While these results are intuitive and straightforward, they help guide modeling, analysis, and intuition for our application of adaptation to climate change in the forestry sector.

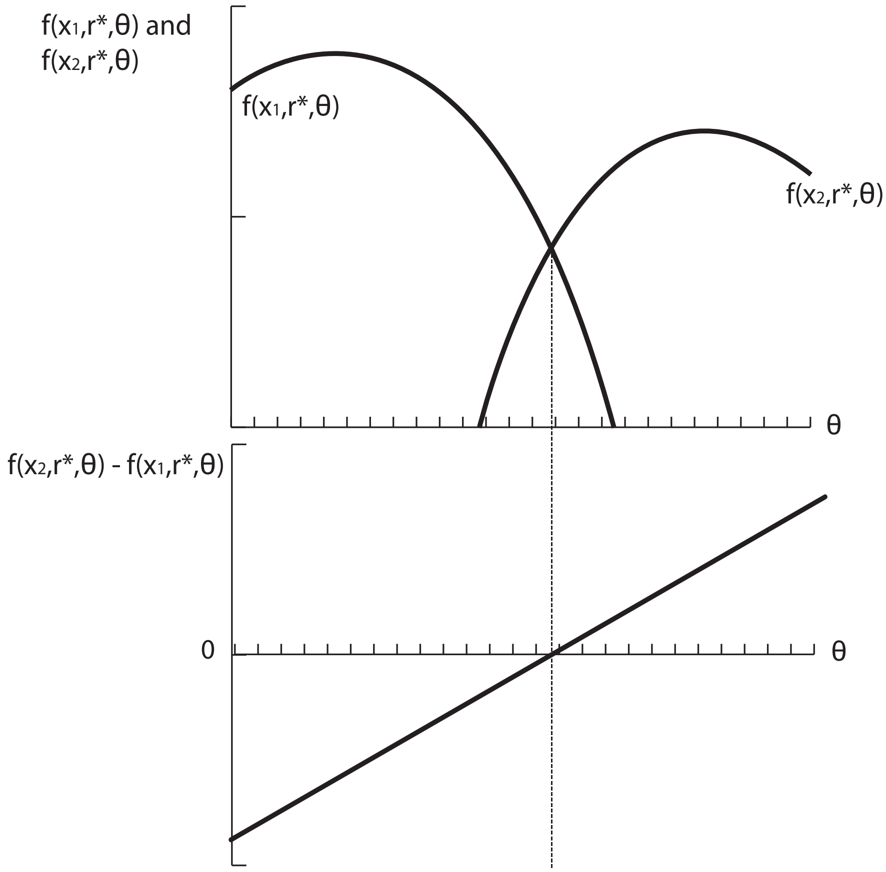

Figure 1: Value of Adaptation on the Extensive Margin

4 Forestry Application under Dynamic Environmental Change

While our simple model generates some useful and provocative insights, the assumption of *marginal* environmental change hardly seems applicable. Large-scale environmental changes, by definition, are non-marginal changes spread over potentially long time periods. Adaptation over millennia to our prevailing climatic conditions is reflected by society's current choices of capital and infrastructure. In our application to climate change, we compare values of adaptation to non-marginal changes across a broad geography, integrated over two centuries. Strict application of our theoretical results predicts that small exogenous changes in our environment may cause us to change our decisions on the intensive margin, but that these adaptations will be relatively inconsequential to economic welfare. Further, even small environmental changes may cause a threshold to be crossed where a completely different discrete choice becomes optimal; this extensive margin adaptation can have high value. In what follows, we generate a fully dynamic case study of private timberlands in

California.

4.1 A Model for Fully-Adaptive Decision-Making

Our decision unit is a single patch sufficient to support a single mature tree. We assume that the forester knows the patch characteristics that inform him of the growth potential for each species at that location. We also assume the forester is forward looking: he can predict how growth, price, and other parameters will change exogenously over time as a result of climate change. The adaptation strategy the forester follows is anticipatory, and we implicitly assume perfect foresight on his part (we will relax this in the next section). For example, he may find it optimal to plant a species today that will be poised to take best advantage of anticipated future growth characteristics of the site even if current conditions do not favor that species. We assume the social value of adaptation is measured by the gain to the private forester; in other words, the individual incentives for adaptation equal the social incentives. Expanding upon the dynamic programming approach of Provencher (1995) and Plantinga (1998), which did not take into account either climate change or extensive margin adaptation, the annual choice facing the forester is between (1) harvesting the current stand and replanting the same species, (2) harvesting the current stand and replanting with a different species, and (3) letting the current stand grow another year. The state variables are the age and species of the tree, and we must explicitly account for calendar time since it maps to parametric changes in tree growth and price, which are driven by climate change.

Our model generalizes (and nests) the classical Faustmann treatment and can be run conditional on any given climate scenario. Calendar year is given by t, the discount factor is δ, harvest cost (per unit volume) is H, and replanting cost is R. The function $P(s,t)$ gives the price per unit volume for species s in year t; time dependence reflects the effects of climate change on global timber output (and therefore price). The function $K(a,t,s)$ gives the volume of a tree of age a, in year t, and of species s.

At some period T in the far distant future, the terminal condition is the value of the stand for an infinite sequence of Faustmann rotations, given constant prices, constant growth rates, and no change in species. At the terminal time the optimal rotation age a^* is calculated from the

Faustmann equation, giving the terminal period value function as follows:

$$J_T(a,s) = [(P(s,T) - H) \times K(a,T,s)(1 + e^{-ra^*} + e^{-2ra^*} + e^{-3ra^*} + \dots)]e^{-r(a^*-a)} \tag{3}$$

The value function for any calendar year t before the terminal period is:

$$J_t(a,s) = \max \begin{cases} (P(s,t) - H) \times K(a,t,s) - R + \delta J_{t+1}(1,s_1), \\ (P(s,t) - H) \times K(a,t,s) - R + \delta J_{t+1}(1,s_2), \\ \vdots \\ (P(s,t) - H) \times K(a,t,s) - R + \delta J_{t+1}(1,s_S), \\ \delta J_{t+1}(a+1,s) \end{cases} \tag{4}$$

With S species, the first S options refer to the profits from harvesting plus the discounted value of a new tree of each species, type $s_1, s_2, \dots, s_S$. The final option is the discounted value of the current tree if it grows another year. For each age (a), species (s), and calendar date (t) the largest of these $S+1$ options is chosen and results in a value for that "state". The control variable is just the choice index from the $S+1$ management options.

4.2 Calculating the Value of Adaptation

A key advantage of this dynamic programming approach is the ability to explicitly model decision-making and adaptation. By storing the choice index for each year, we have a record of the forester's decisions, denoted D^*, to harvest and replant at any point in time from any starting condition. To proceed, we distinguish between the parameters (prices, growth parameters, etc.) defining the state of the world with and without climate change. Let σ_c and σ_w denote these parameter vectors respectively. Then $D^*(\sigma_c)$ is the set of decisions optimized given parameters in a climate change scenario, and $D^*(\sigma_w)$ is the set of decisions optimized given parameters in a world without climate change.

Our fully-adaptive model accounts for adaptation on both the extensive and intensive margins. As noted earlier, much of the literature on forestry has focused only on the intensive margin by only optimizing the rotation period. To isolate the value of adaptation on the intensive margin alone, we add an additional decision set, $D^*_{Int}(\sigma_c)$, where the forester is limited to replanting the same species after harvest and thus is able to adapt only on the intensive margin.

The present value of forest land, denoted $\gamma(\sigma, D)$, depends on the true state of nature (whether climate change exists) and on the adaptive behavior of the landowner (whether rotation interval and/or species mix are optimized for the given state of nature). Table 1 displays the five possible values:

- $\gamma_1 = \gamma(\sigma_w, D^*(\sigma_w))$: value without climate change with decisions optimized without climate change
- $\gamma_2 = \gamma(\sigma_c, D^*(\sigma_w))$: value with climate change with decisions wrongly optimized without climate change
- $\gamma_3 = \gamma(\sigma_c, D^*_{Int}(\sigma_c))$: value with climate change with decisions correctly optimized for climate change only on the intensive margin and not on the extensive margin
- $\gamma_4 = \gamma(\sigma_c, D^*(\sigma_c))$: value with climate change with decisions correctly and fully optimized for climate change on both the extensive and intensive margins
- $\gamma_5 = \gamma(\sigma_w, D^*(\sigma_c))$: value when incorrectly assuming a climate change scenario that does not come to pass but is used as a basis for decision-making.

We can interpret the γ value functions in terms of the f value functions in Appendix A. γ_1 is analogous to $f(x^*(\theta), r^*(\theta), \theta)$. γ_2 is analogous to $f(x^*(\theta), r^*(\theta), \theta + \Delta\theta)$. γ_3 is analogous to $f(x^*(\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta)$. γ_4 is analogous to $f(x^*(\theta + \Delta\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta)$. γ_5 is analogous to $f(x^*(\theta + \Delta\theta), r^*(\theta + \Delta\theta), \theta)$.

Table 1: Possible Values of Forest Land

		Decisions optimized assuming: no Climate Change	Decisions optimized assuming: Climate Change
State of world:	no Climate Change	$\gamma(\sigma_w, D^*(\sigma_w))$	$\gamma(\sigma_w, D^*(\sigma_c))$
	Climate Change	$\gamma(\sigma_c, D^*(\sigma_w))$	$\gamma(\sigma_c, D^*(\sigma_c))$, $\gamma(\sigma_c, D^*_{Int}(\sigma_c))$

Comparing these values will allow us to predict both the impact of climate change and the value of adaptation in offsetting this impact. Figure 2 graphically illustrates this approach. The horizontal axis represents the environmental parameters affected by climate change. In our forestry example this is a compilation of parameters such as temperature and rainfall. The vertical axis measures the present value of forest land under different adaptive responses to environmental conditions.[1] Here, the difference between $\gamma(\sigma_c, D^*(\sigma_c))$ and $\gamma(\sigma_w, D^*(\sigma_w))$ is the effect of climate change with adaptation. The difference between $\gamma(\sigma_c, D^*(\sigma_w))$ and $\gamma(\sigma_w, D^*(\sigma_w))$ is the effect of climate change without adaptation. The difference between $\gamma(\sigma_c, D^*(\sigma_c))$ and $\gamma(\sigma_c, D^*(\sigma_w))$ is the total value of adaptation.

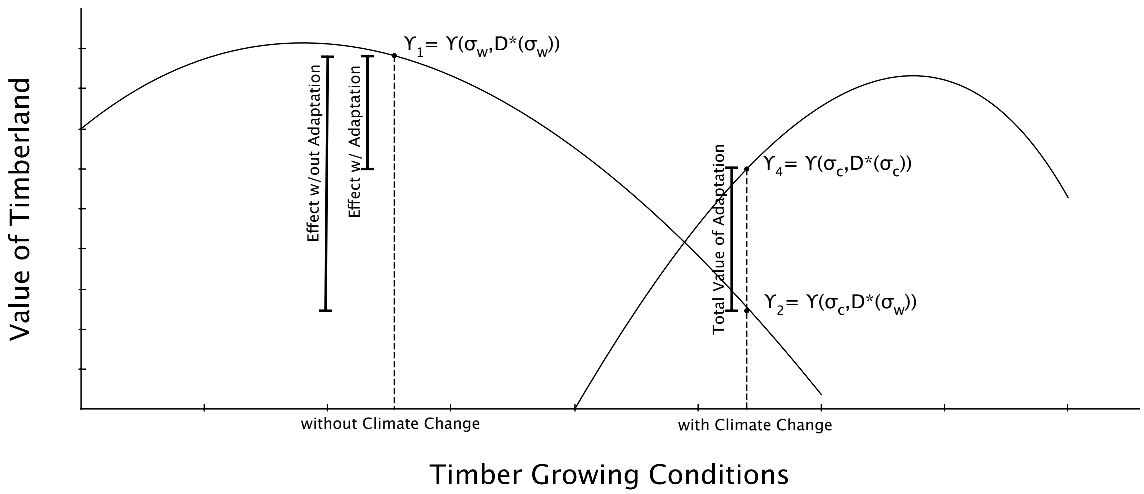

Figure 2: Vertical braces represent differences in value due to climate change

To maintain consistency with the theoretical model, we examine the *marginal* value of adaptation, which (for both the extensive and intensive margins combined) is given by $\lim_{\Delta\sigma \to 0} \frac{\gamma_4 - \gamma_2}{\Delta\sigma}$. Here $\Delta\sigma$ represents the change in site-level characteristics arising from climate change. The extensive margin component is $\lim_{\Delta\sigma \to 0} \frac{\gamma_4 - \gamma_3}{\Delta\sigma}$, and the intensive margin component is $\lim_{\Delta\sigma \to 0} \frac{\gamma_3 - \gamma_2}{\Delta\sigma}$. Because climate change in our application is a non-marginal change, our theoretical results may only hold approximately. We will use our model to estimate the value of adaptation to this discrete change. The total value of adaptation on the extensive margin can be calculated directly by $\gamma_4 - \gamma_3$ and the total value of adaptation on the intensive margin by $\gamma_3 - \gamma_2$. The remainder of this application calculates these quantities for California's timber industry.

[1]An additional extensive adaptation such as alternative land use could be modeled by adding a floor to the value of timberland, represented as a horizontal line in Figure 2. The value of this additional extensive adaptation could still be arbitrarily large due to the difference in the slope at the threshold crossing.

4.3 Adaptation in the California Timber Industry

The timber industry in California is economically and socially important. It dominates public policy over forests that cover 25% of California land area, and the forest products industry is the single largest employer in several counties. However, in past years California timber production has fallen dramatically due to a growing emphasis on recreation, increased wildfires, and threatened species protection. From 1991 to 2006, timber production fell by 45% (Spero 2006). Climate change has the potential to further impact the timber industry. While warming and increased atmospheric carbon dioxide are likely to increase forest productivity, altered hydrology associated with changing precipitation patterns in combination with increasing climate stress may reduce growth and harvest potential. On top of drier conditions, reduced snowpack, and earlier snowmelt, wildfires, invasive species, and extreme weather events are examples of critical stress factors that might be amplified by climate change. Individual site characteristics and projected climate change will determine the direction and magnitude of change. In addition, climate change affects include forest dieback, shifts in species ranges, ecosystem composition, and changes in global timber prices, all of which must be accounted for in a structural model of adaptive decision making.

In the face of climate change, a declining timber industry, and market forces, foresters and landowners will adapt their behavior, and this has the potential to drive major changes in timber land use. Timber production must compete with expanding housing in ex-urban areas and demand for land for other uses. For example, wine production on the Mendocino coast has surpassed timber as the top source of revenue in the county. There is much evidence in both the ecosystem management literature and the environmental economics literature of anticipatory adaptation to climate change by forest managers. A planted forest allows for a major set of adaptations as the decision to plant depends of location, species, stock quality, and other factors. In the ecosystem management literature, reducing the rotation age followed by replanting to speed the establishment of better-adapted forest types is commonly referenced as an adaptive action (Spittlehouse and Stewart 2003; Linder et al. 2000). In fact, forest managers have already begun to anticipate climate change in their management decisions. In southern Finland, managers are changing the species composition to form more stable forests by switching from spruce and pine to birch. Sweden has been working on a breeding program in an attempt to develop trees that would be suitable to projected future climate. In British Columbia, climate change strategies have been introduced to proactively combat the projected increase in fires and insects due to climate change (Roberts 2009). Finally, forest managers are eagerly planning to participate in emerging markets for carbon offsets, which would further change existing management plans.

Our application includes all sites in California classified as "privately owned" and "working" timberland by the California Department of Forestry. We examine 687 sites across the state, at a size of about 16,000 hectares each. The biological model mapping climate scenarios into site-specific growth parameters is from Hannah et al. (2012) who coupled a California-specific ecological model of tree productivity and a global timber price model in order to calculate changes in statewide timber value under various climate and management scenarios.

The inputs into our adaptive decision-making model include species-specific growth rate parameters and timber prices, all of which vary spatially due to geographic heterogeneity in growth conditions and over time due to climate change. The ecological model of tree productivity takes site-specific information on climate conditions (temperature and precipitation), soil conditions, and species physiological characteristics to generate growth curves that fit the functional form: $Volume(age) = e^{\alpha - \frac{\beta}{age}}$. The growth rate parameters α and β are site, species, birth year, and climate scenario dependent (Hannah et al. 2012). We also adopt species-specific annual price projections under baseline and climate change scenarios from Hannah et al. (2012), which were originally derived from global timber price projections (Sohngen et al. 2001; Hannah et al. 2012). Internal consistency is satisfied because Sohngen, Mendelsohn, and Sedjo (2001) account for adaptation in their global model, and furthermore, we assume whether or not adaptation occurs within California's relatively small market has no impact on global prices.

Implemented in MATLAB, we solve our recursive problem one site at a time, with each site optimized independently. In order to allow for heterogeneity of tree species on each site, we generalize our decision unit from a single representative tree to a stand of trees consisting of any possible mix of species. Every year, we find the values across an age range from 0 to 100 years old and across four species (Douglas-Fir, Ponderosa Pine, Redwood, and Hemlock). These four species account for over 92% of harvest value on private lands in California (Hannah et al. 2012). While both Douglas-Fir and Ponderosa Pine share similar prices, Redwood is 136% more valuable, but it can only be grown along California's north coast and some select areas of southern coastal Oregon.

Hemlock, while not as strong as Douglas-Fir when used for structural applications, is an abundant and versatile softwood species with a price 54% less than that of Douglas-Fir and Ponderosa Pine. It is also noteworthy that in addition to replanting various species trees, our model allows us to generalize to any change in land use such as conversion from forestry to other economic sectors (residential, industrial, agricultural), although we focus on within-industry analysis.

Key assumptions in our model adopted from Hannah et al. (2012) include discount rate (a constant 5%), replanting cost ($988 per hectare), and harvest cost ($31 per cubic meter). Following our assumption that forest managers are rational profit-maximizers, the initial species for each site in our model is the most profitable species given the no climate change scenario.

To facilitate interpretation of our main results, we normalize all adaptation value calculations by the status quo value, γ_2. Thus the reported statistics can be interpreted as the total value of adaptation as a fraction of the value of the forest[2]. We can express the combined increase in value from adaptation in percentage terms as $\frac{\gamma_4-\gamma_2}{\gamma_2}$. This term can be separated into the percentage increase from the extensive margin and the percentage increase from the intensive margin: $\frac{\gamma_4-\gamma_2}{\gamma_2} = \frac{\gamma_4-\gamma_3}{\gamma_2} + \frac{\gamma_3-\gamma_2}{\gamma_2}$.

[2]Technically we normalize by $\frac{\gamma_2}{\Delta\sigma}$,which is constant across intensive vs. extensive adaptation, so the percentage value of adaptation on the extensive margin (for example) is $\frac{\gamma_4-\gamma_3}{\Delta\sigma} * \frac{\Delta\sigma}{\gamma_2} = \frac{\gamma_4-\gamma_3}{\gamma_2}$

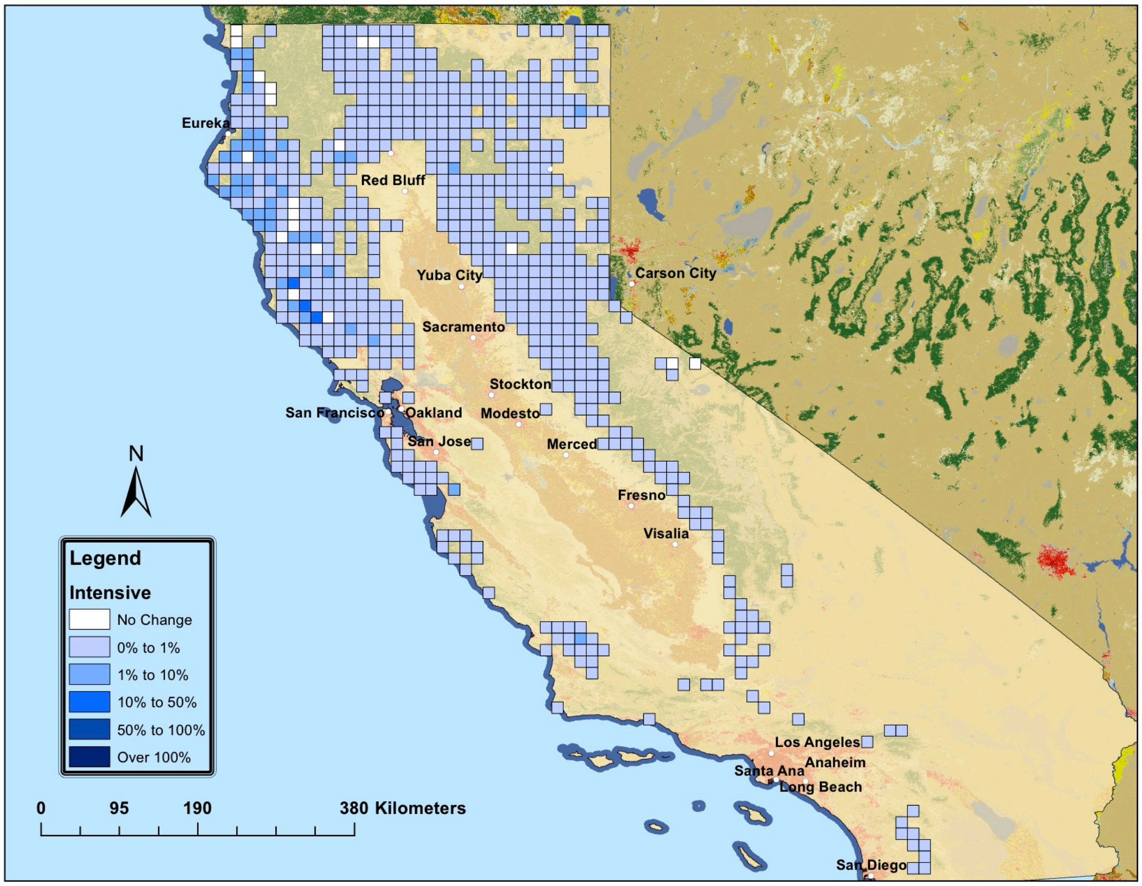

Figure 3: % Change in Value from Adaptation on the Intensive Margin under PCM1 A2

Empirical results from this structural decision-making model accord with our theoretical findings. Our theoretical model suggested that adaptation should occur but that the marginal value of adaptation on the intensive margin would be 0, and therefore the total value of adaptation on that margin should be quite small. In fact intensive margin adaptation to climate change decreases rotation age in 55% of patches and increases rotation age in 28% of patches, but in the map of the economic effects of the intensive margin (Figure 3) two features are apparent. First, there is very little spatial variation in the value of adaptation on the intensive margin. Second, values are very small, typically between 0 and 1% (Figure 4).

Figure 4 shows the extensive margin results, which are also consistent with our theoretical predictions that the marginal value of adaptation on the extensive margin can be quite large (sometimes over 50%) but is typically 0. Figure 5 expresses the threshold effects induced by climate change. Under the PCM1 A2 scenario, 86 sites lie on species habitat boundaries. Climate

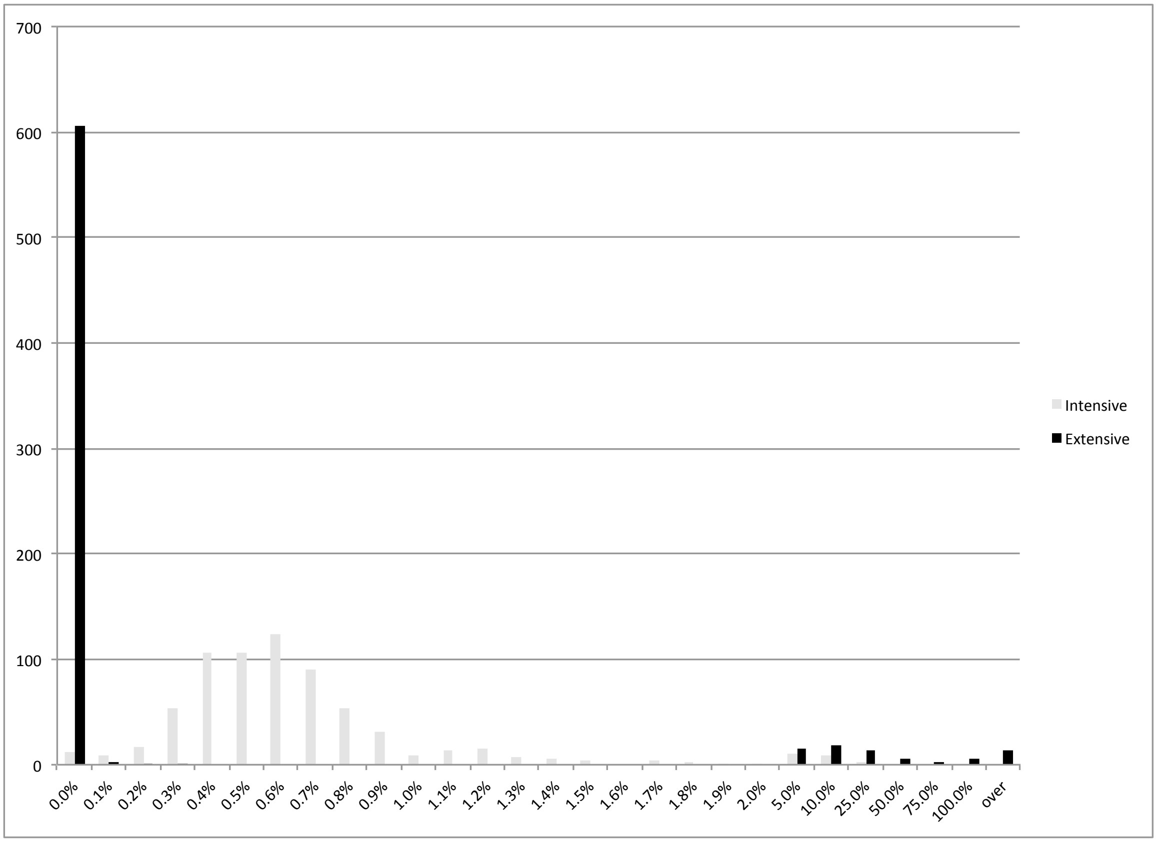

Figure 4: Histogram of % Change in Value from Adaptation on the Extensive and Intensive Margins (PCM1 A2)

change causes these boundaries to shift over the duration of our study so more valuable species become viable and can replace less profitable species. For example, consider a site located near Ukiah in Mendicino County. In 2000, the most profitable species to grow in that site without the expectation of climate change was Western Hemlock. With climate change under the PCM1 A2 scenario between 2000 and 2080, temperature is estimated to change 3.5 degrees Celsius with little change in precipitation. This change will adversely affect the productivity of Hemlock and, with no adaptation, the value of a hectare of land in that site is predicted to decrease by 32.4% from the baseline value. However, the site has a huge potential to benefit from adaptation on the extensive margin due to the threshold effect of climate change. As temperature increases and species habitat boundaries shift, it is able to grow new species of trees–species that would not have been suitable without climate change, species that are potentially more valuable than the existing Hemlock. One such species is Redwood, which carries a significantly higher price than Hemlock. On the intensive margin, the rotation interval is adjusted by one year, which accounts for a .019% increase in value

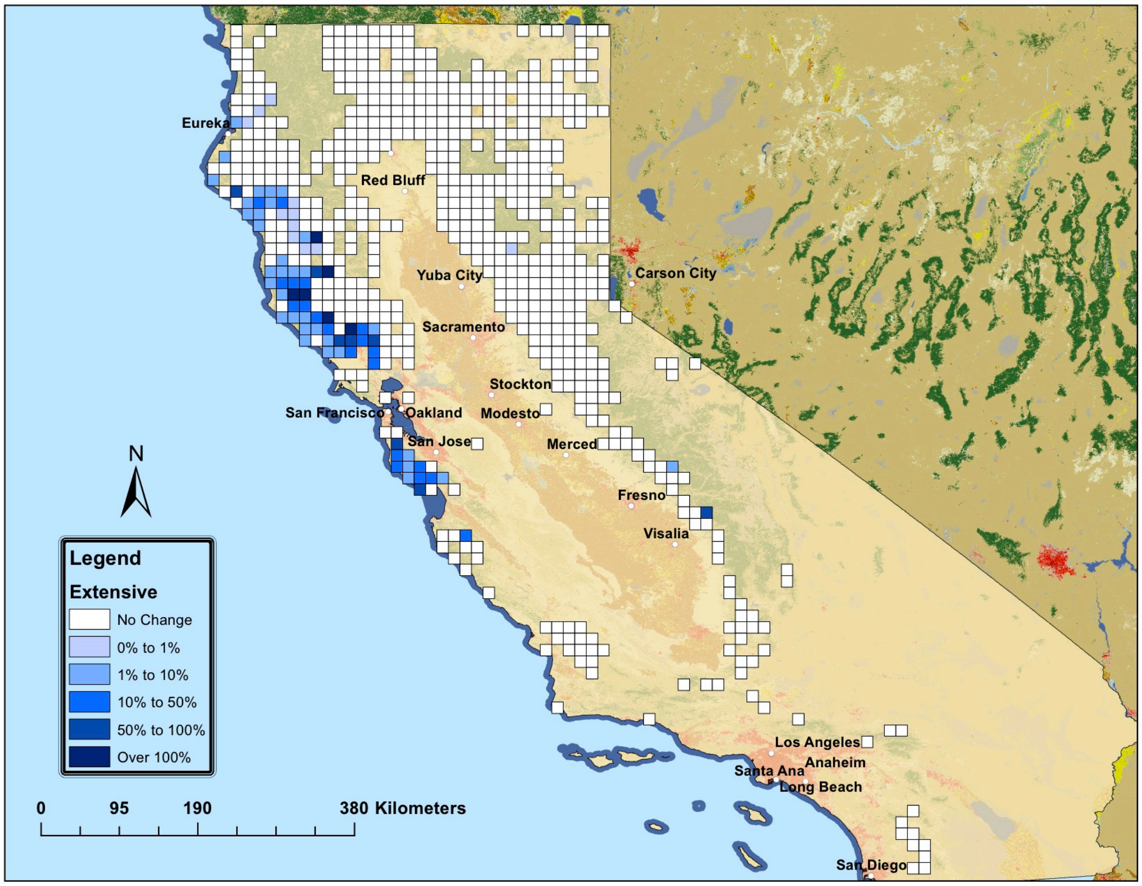

Figure 5: % Change in Value from Adaptation on the Extensive Margin under PCM1 A2

over the no adaptation scenario. On the extensive margin, a shift from Hemlock to Redwood is able to increase the value of land in this site by 276% over the intensive margin only scenario. With adaptation, the most valuable strategy is to let Hemlock grow until 2042 and then replant with Redwood at a 53-year rotation interval. Thus full adaptation not only completely mitigates climate change, it is able to turn a 32.4% negative effect into an overall positive effect.

In Table 2, where we focus on the sites where threshold boundaries are located (so species are changed), we see significant increases in economic value on the extensive margin while the intensive margin remains close to zero.

Table 2: % Change in Value from Adaptation in Sites at Threshold Boundaries

Climate Scenario	Extensive Adaptation	Intensive Adaptation	Number of Sites
PCM1 A2	18.9%	0.87%	86

While not the central focus of this analysis, our model also generates an aggregate estimate of the economic consequences of climate change to the forestry sector in California. Figure 6 shows the net overall effect of climate change with adaptation on the forestry sector given the PCM1 A2 scenario. The aggregate effect of climate change with adaptation for the entire state in this scenario is to decrease the value of timberland by 10.3%. Using net present value statewide timber revenue assessments from Hannah et al. (2012), which estimates California timber to be a $27 billion industry, this translates to an absolute loss of $2.8 billion.

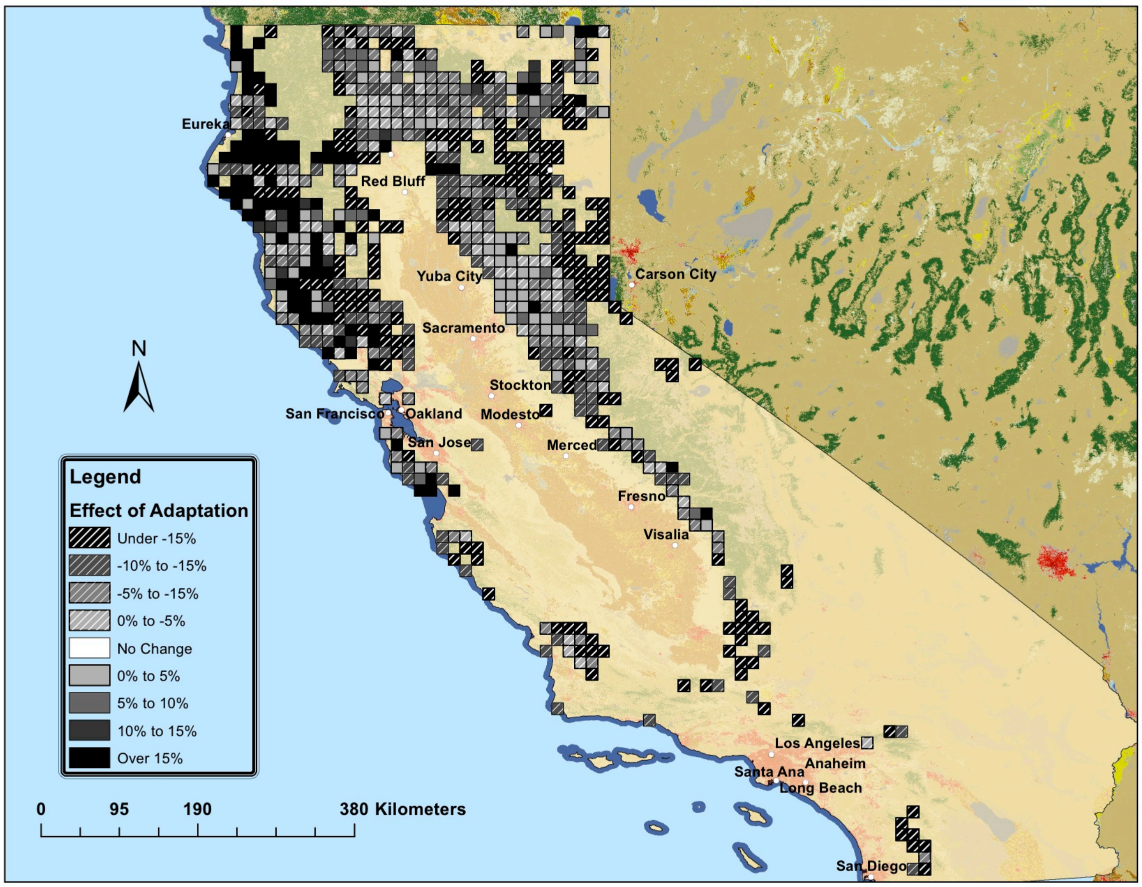

Figure 6: Effect of Climate Change with Adaptation

4.4 Adaptation with Stochastic Timber Prices and Learning

Thus far our analysis has assumed that prices are a deterministic function of time and the climate scenario. In reality, while general trends may be predictable, there is still considerable uncertainty about how global markets will respond to climate change. We argue that this uncertainty will be resolved over time as the effects of climate change are increasingly realized. In order to build intuition on the effect of stochastic prices on the value of adaptation, we extend our model to allow for stochastic prices and adaptive learning. Deterministic price projections from global timber models exist under various climate scenarios (Sohngen et al. 2001). Learning will occur over time, but will likely be corrupted by random price shocks that preclude instantaneous revelation of the true price path. Importantly, since California is a small player in a global timber market, while global timber price projections intrinsically account for adaptation, adaptive behavior within California will not influence global prices. In this setting, suppose we have two stochastic price series, one with and one without climate change. In the baseline scenario without climate change prices are given by:

$$P^w(s,t) = \chi_t^w(\bar{P}_t^w - \bar{P}_0^w)F_s + \bar{P}_0^w F_s \tag{5}$$

where χ_t^w is a random variable, $\bar{P}_t^w$ is the deterministic global timber price projection in year t, and F_s is the species-specific price adjustment factor. The random shock χ_t^w is drawn from a normal distribution with mean Φ^w and known variance $1/\zeta$ (precision ζ). The mean, Φ^w is unknown, but the forester has a prior distribution over Φ^w that is normally distributed with a mean μ_t^w and precision τ_t^w.

Prices under the climate change scenario are given by:

$$P^c(s,t) = \chi_t^c(\bar{P}_t^c - \bar{P}_0^c)F_s + \bar{P}_0^c F_s \tag{6}$$

where $\chi_t^c \sim N(\Phi^c, \frac{1}{\zeta})$ and the prior beliefs over $\Phi^c \sim N(\mu_t^c, \frac{1}{\tau_t^c})$.

Random noise (characterized by ζ) can observed from historical means and standard deviations of the price distributions for saw-timber (Buongiorno et al. 1994; Reeves and Haight 2000). For this analysis we assume $\zeta = 16$, which corresponds to a 25% standard deviation and accords with data from the literature on fluctuating prices (Brazee and Mendelsohn 1988). Because our best initial beliefs of each price series are simply the deterministic price projections, $\mu_0^w = \mu_0^c = 1$. Including a separate species-specific price adjustment factor allows for the variance of the price to scale across species. Learning occurs by repeatedly observing χ_t each year, and by standard Bayesian manipulation of conjugate distributions, the updating formulae for the belief hyperparameters are:

$$\mu_{t+1} = \frac{\tau_t \mu_t + \zeta \chi_t}{\tau_t + \zeta} \tag{7}$$

$$\tau_{t+1} = \tau_t + \zeta \tag{8}$$

Because the precision increases by the same amount deterministically each year, this model of uncertainty and learning in price trajectories requires just one additional state variable, μ_t. Thus, given stochastic prices and learning, the stochastic dynamic programming equation is:

$$J_t(a, s, \mu_t) = \max \begin{cases} (E[P(s,t)|\mu_t] - H) \times K(a,t,s) - R + \delta E[J_{t+1}(1, s_1, \mu_{t+1})], \\ (E[P(s,t)|\mu_t] - H) \times K(a,t,s) - R + \delta E[J_{t+1}(1, s_2, \mu_{t+1})], \\ \vdots \\ (E[P(s,t)|\mu_t] - H) \times K(a,t,s) - R + \delta E[J_{t+1}(1, s_S, \mu_{t+1})], \\ \delta E[J_{t+1}(a+1, s, \mu_{t+1})] \end{cases} \tag{9}$$

Under this revised (and generalized) model, we can first isolate the effect of price uncertainty on the value of adaptation. If prices are stochastic but no learning occurs, the value of adaptation is the same as in the deterministic case. Equation (7)[3] shows that the updated μ_{t+1} is a weighted average of the prior belief and a new observation of χ_t. When the precision of beliefs τ_t becomes very large relative to ζ, the forester is very certain of Φ and the updating formula approaches $\mu_{t+1} = \mu_t$. Consequently, with no updating and remembering $\mu_0 = ... = \mu_t = \mu_{t+1} = 1$, the price shock is integrated out of the value function at every stage, leaving the same value function, policy function, and value of adaptation as in the deterministic case.

Adding both learning and stochastic prices affects the two margins of adaption differently in our study. First, the absolute value of adaptation on the extensive margin is unaffected by learning. This is a consequence of the fact that the species-specific price adjustment factor F_s is invariant

[3]No Learning was solved using $\tau_0 = 1,000,000$. With Learning, τ_0 can be derived from comparing the variance in prices across different scenarios, and in this application, we assume $\tau_0 = 100$.

over time. In order to keep the number of dimensions of uncertainty reasonable, we assume while climate change will effect global timber prices, the ratio of relative prices across species is constant. Therefore, learning about the overall price path of timber will not result in any additional value from switching species.

Second, the absolute value of adaptation on the intensive margin increases, driving a percentage increase in the value on the intensive margin and a percentage decrease in value on the extensive margin. When there are fixed costs, such as harvesting and replanting costs, the solution to the stochastic tree-cutting problem will depend on timber prices (Hildebrandt and Knoke 2011). In our model, prices are a function of the unknown mean Φ, which is revealed over time by learning. A general result from Fisher and Hanemann (1984), which is also leveraged by Plantinga (1998), uses Jensen's inequality and convexity of the maximum operator to demonstrate the option value in waiting to acquire additional information about an unknown variable, when that variable provides useful information about the investment. On the intensive margin, the optimal rotation interval depends on the price path. The mean parameter Φ is unknown in the current period t but will be revealed in the future; after learning occurs, an optimal choice can be made. Without learning, the forester also does not know Φ, but must always act under uncertainty about Φ in all periods. This option value result reflects the value of forthcoming information about Φ. This third result is consistent with predictions from the literature on forestry option pricing models given stochastic prices, and for a handful of sites, we observe a longer rotation interval compared to the no learning policy. Table 3 shows the numerical results under the model of uncertainty and learning. As is predicted by the above results, the value of adaptation on the extensive margin is unaffected by learning. On the intensive margin, the value of adaptation increases with learning. The final two columns display the same results as fractions of the default value of the forest.

Table 3: Value of Adaption with & without learning under PCM1 A2 climate scenario

	Absolute Value Ext. $(\gamma_4 - \gamma_3)$	Absolute Value Int. $(\gamma_3 - \gamma_2)$	% Value Ext. $\frac{(\gamma_4-\gamma_3)}{\gamma_2}$	% Value Int. $\frac{(\gamma_3-\gamma_2)}{\gamma_2}$
No Learning	1.210×10^9	7.444×10^7	18.9%	.87%
Learning	1.210×10^9	9.385×10^7	18.1%	1.1%

We have focused on stochastic timber prices, but this is just one way to incorporate uncertainty into an optimal rotation problem. There may also be uncertainty about future effects on timber yields, and we could have chosen alternatively to model adaptive learning about the proper growth parameters either in addition to or in place of learning about the price path. In both cases, we speculate an increase in the value of adaptation on the intensive margin would reflect a similar option value result, while choices in how to model learning would drive extensive margin results. For example, if species-specific learning occurred concerning the effect of climate change on individual timber species yields, there could be a significant change in the absolute value of adaptation on the extensive margin due to learning. Furthermore, we use historical timber prices to parameterize the stochastic price process, assuming the past is our best guide to the future, but one could imagine prices becoming more volatile in the future as a result of climate change. One way this might occur is through greater random noise and price shocks due to extreme weather events (in our model, a decrease in the precision ζ), which would make it more difficult to learn the true price path, thus decreasing the effect of learning. Finally, another possible way to incorporate uncertainty and learning would be to relax the assumption of a known precision ζ and have priors and learning about both an unknown mean and an unknown precision for the random shock χ_t.

5 Conclusion

Behavioral adaptation to exogenous change is both implicit and fundamental, and private forestry is an ideal sector in which to model dynamic adaptive responses to climate change. It follows basic economic intuition that ignoring adaptation may lead to serious mis-estimation of climate change impacts. We make a theoretical distinction between adaptation on the extensive versus intensive margin and develop a stylized model to gain general insights. We are able to make sharp analytical predictions: on the extensive margin the marginal value of adaptation can be arbitrarily large while on the intensive margin the marginal value of adaptation will be zero. Finally, recognizing the full dynamic complexities of modeling different types of adaptation to large-scale environmental change, we build it back up for a climate climate application using a structural dynamic model for adaptive decision-making, revisiting a classic forestry economics problem with a dynamic programming approach that incorporates uncertainty and learning.

Our initial naive expectation was that the bias introduced from ignoring adaptation would always be economically significant. However, we found both theoretically and in our application that this is not always the case. Indeed, if a threshold is crossed, then there could be a significant value of adaptation on the extensive margin, which would cause the welfare estimate to differ substantially in the adaptation versus no adaptation scenarios. However, if no threshold is crossed, then the value of adaptation on both margins is negligible, so the welfare estimate is likely to be reasonably estimated even without taking into account adaptation. This insight alters the lens through which researchers and policy makers evaluate welfare effects of environmental change. This is the first study to examine adaptation and forest rotation in a Bayesian framework and, while adding uncertainty and learning to the model generates some interesting insights, our basic conclusions about the value of adaptation are maintained.

This framework for assessing the value adaptation can extend well beyond climate change to study adaptation in the electricity industry, coastal management in response to rising sea levels, groundwater management, public health, and other timely public policy dilemmas. Regardless of application, we anticipate that the basic general insight uncovered here–that adaptation on the intensive margin will have negligible economic value, but that adaptation on the extensive margin may have large economic consequences–will endure.

Appendix A. A Formal Treatment of the Stylized Model

Suppose an economic agent wishes to maximize a function g where r is a choice variable and θ is a parameter:

$$\begin{aligned} V(\theta) &= \max_r g(r, \theta) \\ &= g(r^*(\theta), \theta) \end{aligned} \tag{10}$$

To illustrate environmental change, consider a small exogenous change in the parameter θ. This change affects the value function $V(\theta)$ both directly and indirectly through the choice $r(\theta)$. A total derivative $\frac{dV}{d\theta}$ takes into account both these effects:

$$\frac{dV}{d\theta} = \frac{\partial g}{\partial r}\frac{\partial r}{\partial \theta} + \frac{\partial g}{\partial \theta} \tag{11}$$

Referring to Equation (1), the full effect with adaptation ($\frac{dV}{d\theta}$) is composed of the value of adaptation ($\frac{\partial g}{\partial r}\frac{\partial r}{\partial \theta}$) plus the effect without adaptation ($\frac{\partial g}{\partial \theta}$). Simple application of the Envelope Theorem takes advantage of the fact that $\frac{\partial g}{\partial r} = 0$ since the choice of r is optimal. Here adaptation does occur (i.e. $\frac{\partial r}{\partial \theta} \neq 0$), but the value of adaptation is zero ($\frac{\partial g}{\partial r}\frac{\partial r}{\partial \theta} = 0$). Essentially, because agents behaved optimally prior to the exogenous change, adaptation to a small change has no economic value. *Here ignoring adaptation has no effect on estimates of the economic impacts of the exogenous change.* This excruciatingly simple insight suggests that small structural shifts in the environment will be responded to by economic agents, but will not have appreciable economic consequences.

To accommodate the extensive margin, we will draw on recent literature that derives envelope theorems for use with discrete and arbitrary choice sets (Milgrom and Segal 2002; Sah and Zhao 1998). We assume that various functions f are all strictly concave in θ and r and differentiable in θ and r and that the envelope function V is continuous. Given these conditions, V is differentiable almost everywhere. We will draw new envelope theorem conclusions from a hybrid optimization with both discrete and continuous choice variables:

$$
\begin{aligned}
V(\theta) &= \max_{x,r} f(x,r,\theta) \\
&= f(x^*(\theta), r^*(\theta), \theta)
\end{aligned}
\tag{12}
$$

where x denotes the discrete choice variable and r denotes the continuous choice variable. The optimal choices of both depend on the parameter θ, and they are determined jointly, conditional on each other. $x \in X$ where, without loss of generality, the choice set $X = \{x_1, x_2\}$. $x^*(\theta)$ denotes the values of x that are optimal for a given value of θ:

$$
x^*(\theta) = \begin{cases} x_1 \text{ if } f(x_1, r^*(\theta), \theta) \geq f(x_2, r^*(\theta), \theta) \\ x_2 \text{ if } f(x_1, r^*(\theta), \theta) < f(x_2, r^*(\theta), \theta) \end{cases} \tag{13}
$$

We assume x is a monotonic function of θ, and as seen from Figure 7, the optimal choice correspondence is given by:

$$
x^*(\theta) = \begin{cases} x_1 \text{ if } \theta \leq \theta_2 \\ x_2 \text{ if } \theta > \theta_2 \end{cases} \tag{14}
$$

By Definition 1, θ_2 is the threshold point at which $f(x_1, r^*(\theta_2), \theta_2) = f(x_2, r^*(\theta_2), \theta_2)$.

In a setting where $\Delta\theta$ is imposed exogenously and agents respond optimally, the total change in value can be expressed as:

$$
\Delta V = f(x^*(\theta + \Delta\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta), \theta) \tag{15}
$$

By adding and subtracting the same terms to the right side, we can rewrite this as:

$$
\begin{aligned}
\Delta V = f(x^*(\theta + \Delta\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) \\
+ f(x^*(\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta), \theta + \Delta\theta) \\
+ f(x^*(\theta), r(^*\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta), \theta)
\end{aligned}
\tag{16}
$$

Equation (16) contains 3 terms, which correspond to Equation (1). We refer to the first term on the right-hand side as the total value of adaptation on the extensive margin, and we denote it as follows:

$$TVA_{ext}(\theta, \theta + \Delta\theta) = f(x^*(\theta + \Delta\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) \tag{17}$$

The second term in Equation (16) is the total value of adaptation on the intensive margin and is denoted:

$$TVA_{int}(\theta, \theta + \Delta\theta) = f(x^*(\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta), \theta + \Delta\theta) \tag{18}$$

The third term is the effect without adaptation. To restate Equation (1), the sum of all three elements is the net effect with adaptation. Standardizing across any magnitude of environmental change requires normalizing by $\Delta\theta$. We will show adaptation can have value even for arbitrarily small $\Delta\theta$.

Dividing Equation (16) by $\Delta\theta$ yields the average effect of the change:

$$\begin{aligned}\frac{\Delta V}{\Delta\theta} = &\frac{f(x^*(\theta + \Delta\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta)}{\Delta\theta} \\ &+ \frac{f(x^*(\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta), \theta + \Delta\theta)}{\Delta\theta} \\ &+ \frac{f(x^*(\theta), r^*(\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta), \theta)}{\Delta\theta}\end{aligned} \tag{19}$$

Applying our previous reasoning to the three terms on the right-hand side of Equation (19) and taking the limit as $\Delta\theta$ approaches 0, we denote the marginal value of adaptation on the extensive margin as:

$$MVA_{ext}(\theta, \theta + \Delta\theta) = \lim_{\Delta\theta \to 0} \frac{f(x^*(\theta + \Delta\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta)}{\Delta\theta} \tag{20}$$

The marginal value of adaptation on the intensive margin is denoted:

$$MVA_{int}(\theta, \theta + \Delta\theta) = \lim_{\Delta\theta \to 0} \frac{f(x^*(\theta), r^*(\theta + \Delta\theta), \theta + \Delta\theta) - f(x^*(\theta), r^*(\theta), \theta + \Delta\theta)}{\Delta\theta} \tag{21}$$

The combined marginal value of adaptation is simply the sum:

$$\begin{aligned} MVA_{combined}(\theta, \theta+\Delta\theta) &= MVA_{ext}(\theta, \theta+\Delta\theta) + MVA_{int}(\theta, \theta+\Delta\theta) \\ &= \lim_{\Delta\theta \to 0} \frac{f(x^*(\theta+\Delta\theta), r^*(\theta+\Delta\theta), \theta+\Delta\theta) - f(x^*(\theta), r^*(\theta), \theta+\Delta\theta)}{\Delta\theta} \end{aligned} \tag{22}$$

Thus the combined marginal value of adaptation over a discrete interval $\Delta\theta$ is the difference between a "smart" agent who adjusts x and r in response to $\Delta\theta$ and a "dumb" agent who experiences the same $\Delta\theta$ but does not adjust x or r.

Figure 7 graphs the value function $V(\theta)$ as the upper envelope of the individual functions $f(x, r, \theta)$ given an optimized level of r for some parameter θ. θ_0 is the "threshold point," because at this point $f(x_1, r^*(\theta_0), \theta_0) = f(x_2, r^*(\theta_0), \theta_0)$. This payoff is depicted as Point A. Point B corresponds to the no extensive margin adaptation payoff $f(x_1, r^*(\theta_0+\Delta\theta), \theta_0+\Delta\theta)$. Point C corresponds to adaptation payoff $f(x_2, r^*(\theta_0+\Delta\theta), \theta_0+\Delta\theta)$. The difference C-B is the total value of adaptation on the extensive margin.

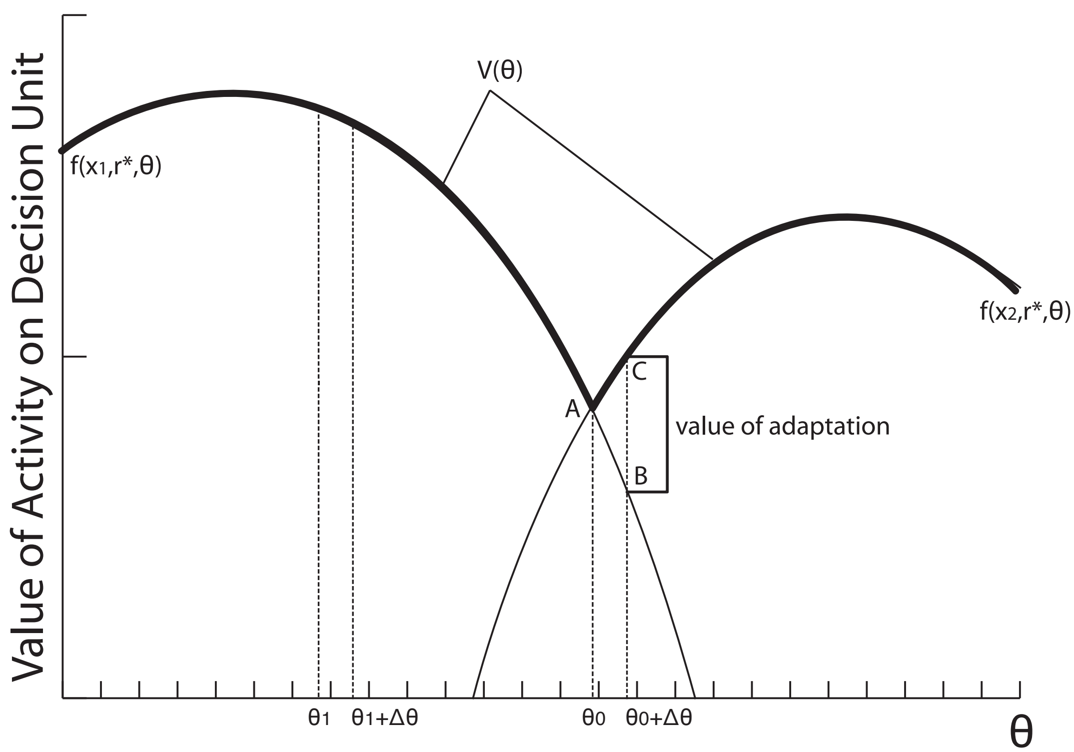

Figure 7: Envelope Function

Proposition 1

Proof. As defined in Equation (21), the marginal value for adaptation on the intensive margin $MVA_{int} = \frac{f(x^*(\theta),r^*(\theta+\Delta\theta),\theta+\Delta\theta)-f(x^*(\theta),r^*(\theta),\theta+\Delta\theta)}{\Delta\theta}$. When $\Delta\theta$ approaches 0, at the limit, this equals $\frac{\partial f}{\partial r}\frac{\partial r}{\partial \theta}$ because $r(\theta)$ is continuous and differentiable. The first-order condition $\frac{\partial f}{\partial r} = 0$ implies the marginal value for adaptation on the intensive margin is 0. ■

On the other hand, the marginal value of adaptation on the extensive margin may behave differently due to the discrete nature of the choice variable and depending on whether the change in θ causes a threshold to be crossed.

Proposition 2

Proof. Suppose a threshold is not crossed in a change from θ_1 to $\theta_1 + \Delta\theta$. Therefore $x^*(\theta_1) = x^*(\theta_1 + \Delta\theta) = x_1$. As defined in Equation (20), the marginal value for adaptation on the extensive margin: $MVA_{ext} = \frac{f(x^*(\theta_1+\Delta\theta),r^*(\theta_1+\Delta\theta),\theta_1+\Delta\theta)-f(x^*(\theta_1),r^*(\theta_1+\Delta\theta),\theta_1+\Delta\theta)}{\Delta\theta} = \frac{f(x_1,r^*(\theta_1+\Delta\theta),\theta_1+\Delta\theta)-f(x_1,r^*(\theta_1+\Delta\theta),\theta_1+\Delta\theta)}{\Delta\theta} = 0$ ■

Proposition 3

Proof. Suppose there is a change from θ_0 to $\theta_0 + \Delta\theta$.

Case 1 *If $\Delta\theta > 0$, then at the threshold point, for any $\Delta\theta$, $x^*(\theta) \neq x^*(\theta + \Delta\theta)$ by Definition 1. Then according to Equation (14), $x^*(\theta_0) = x_1$ and $x^*(\theta_0 + \Delta\theta) = x_2$. The marginal value of adaptation on the extensive margin $MVA_{ext} = \frac{f(x^*(\theta_0+\Delta\theta),r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x^*(\theta_0),r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)}{\Delta\theta} = \frac{f(x_2,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_1,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)}{\Delta\theta} \neq 0$.*

Furthermore, as $\Delta\theta$ approaches 0, $\lim_{\Delta\theta\to 0+} MVA_{ext} = \lim_{\Delta\theta\to 0+} \frac{f(x_2,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_1,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)}{\Delta\theta}$.

Adding and subtracting the same term to the right side,

$$\lim_{\Delta\theta\to 0+} MVA_{ext} = \lim_{\Delta\theta\to 0+} \frac{f(x_2,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_1,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_1,r^*(\theta_0),\theta_0)+f(x_1,r^*(\theta_0),\theta_0)}{\Delta\theta}.$$

By applying the definition of the threshold point at θ_0,

$$\lim_{\Delta\theta\to 0+} MVA_{ext} = \lim_{\Delta\theta\to 0+} \frac{f(x_2,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_1,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_2,r^*(\theta_0),\theta_0)+f(x_1,r^*(\theta_0),\theta_0)}{\Delta\theta}.$$

We can then rearrange these terms to get:

$$\lim_{\Delta\theta\to 0+} MVA_{ext} = \lim_{\Delta\theta\to 0+} \frac{f(x_2,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_2,r^*(\theta_0),\theta_0)}{\Delta\theta} - \lim_{\Delta\theta\to 0+} \frac{f(x_1,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_1,r^*(\theta_0),\theta_0)}{\Delta\theta}.$$

Adding and subtracting to both terms on the right side,

$$\lim_{\Delta\theta\to 0+} MVA_{ext} = \lim_{\Delta\theta\to 0+} \frac{f(x_2,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_2,r^*(\theta_0),\theta_0)-f(x_2,r^*(\theta_0+\Delta\theta),\theta_0)+f(x_2,r^*(\theta_0+\Delta\theta),\theta_0)}{\Delta\theta} - \lim_{\Delta\theta\to 0+} \frac{f(x_1,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_1,r^*(\theta_0),\theta_0)-f(x_1,r^*(\theta_0+\Delta\theta),\theta_0)+f(x_1,r^*(\theta_0+\Delta\theta),\theta_0)}{\Delta\theta}.$$

Again, rearranging each term,

$$\lim_{\Delta\theta\to 0+} MVA_{ext} = \lim_{\Delta\theta\to 0+} \frac{f(x_2,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_2,r^*(\theta_0+\Delta\theta),\theta_0)+f(x_2,r^*(\theta_0+\Delta\theta),\theta_0)-f(x_2,r^*(\theta_0),\theta_0)}{\Delta\theta} - \lim_{\Delta\theta\to 0+} \frac{f(x_1,r^*(\theta_0+\Delta\theta),\theta_0+\Delta\theta)-f(x_1,r^*(\theta_0+\Delta\theta),\theta_0)+f(x_1,r^*(\theta_0+\Delta\theta),\theta_0)-f(x_1,r^*(\theta_0),\theta_0)}{\Delta\theta}.$$

At the limit,

$$\lim_{\Delta\theta\to 0+} MVA_{ext} = [f_\theta(x_2,r^*(\theta_0),\theta_0) + f_r(x_2,r^*(\theta_0),\theta_0)] - [f_\theta(x_1,r^*(\theta_0),\theta_0) + f_r(x_2,r^*(\theta_0),\theta_0)]$$

Applying the first order condition $\frac{\partial f}{\partial r} = 0$,

$$\lim_{\Delta\theta\to 0+} MVA_{ext} = f_\theta(x_2,r^*(\theta_0),\theta_0) - f_\theta(x_1,r^*(\theta_0),\theta_0)$$

Therefore as long as the difference in the slopes of the f functions is greater than 0, the marginal value of adaptation on the extensive margin is positive even when the change in the parameter θ approaches 0.

Case 2 *If $\Delta\theta < 0$. Then according to Equation* (14), $x^*(\theta_0) = x_1$ *and* $x^*(\theta_0+\Delta\theta) = x_1$. *Therefore, a threshold is not crossed and Proposition 2 applies.*

■

References

Adams, R., B. Hurd, S. Lenhart, and N. Leary (1998). Effects of global climate change on agriculture: an interpretative review. *Climate Research 11*(1), 19–30.

Adams, R., C. Rosenzweig, R. Peart, J. Ritchie, B. McCarl, J. Glyer, R. Curry, J. Jones, K. Boote, and L. Allen (1990). Global climate change and us agriculture.

Adams, R. M. (1989). Global climate change and agriculture: An economic perspective. *American Journal of Agricultural Economics 71*(5), pp. 1272–1279.

Alvarez, L. and E. Koskela (2007). Optimal harvesting under resource stock and price uncertainty. *Journal of Economic Dynamics and Control 31*(7), 2461–2485.

Besley, T. and A. Case (1993). Modeling technology adoption in developing countries. *The American Economic Review 83*(2), 396–402.

Brazee, R. and R. Mendelsohn (1988). Timber harvesting with fluctuating prices. *Forest Science 34*(2), 359–372.

Brennan, M. and E. Schwartz (1985). Evaluating natural resource investments. *Journal of business*, 135–157.

Bullard, J. and J. Duffy (1998). A model of learning and emulation with artificial adaptive agents. *Journal of Economic Dynamics and Control 22*(2), 179 – 207.

Buongiorno, J., S. Dahir, H. Lu, and C. Lin (1994). Tree size diversity and economic returns in uneven-aged forest stands. *Forest Science 40*(1), 83–103.

Clarke, H. and W. Reed (1989). The tree-cutting problem in a stochastic environment:: The case of age-dependent growth. *Journal of Economic Dynamics and Control 13*(4), 569–595.

Costinot, A., L. Oldenski, and J. E. Rauch (2009). Adaptation and the Boundary of Multinational Firms. *SSRN eLibrary*.

DeBondt, W. F. and R. H. Thaler (1994). Financial Decision-Making in Markets and Firms: A Behavioral Perspective. *SSRN eLibrary*.

Deschenes, O. and M. Greenstone (2007). Climate Change, Mortality, and Adaptation: Evidence from Annual Fluctuations in Weather in the Us. *SSRN eLibrary*.

Easterling, W. E., P. R. Crosson, N. J. Rosenberg, M. S. McKenney, L. A. Katz, and K. M. Lemon (1993). Paper 2. agricultural impacts of and responses to climate change in the missouri-iowa-nebraska-kansas (mink) region. *Climatic Change 24*, 23–61.

Fankhauser, S. (1994). The social costs of greenhouse gas emissions: An expected value approach. *The Energy Journal 15*(2), 157–184.

Fisher, A. and W. Hanemann (1984). Option value and the extinction of species. *Working paper-Giannini Foundation of Agricultural Economics, California Agricultural Experiment Station*.

Foster, A. D. and M. R. Rosenzweig (1995). Learning by doing and learning from others: Human capital and technical change in agriculture. *The Journal of Political Economy 103*(6), 1176–1209.

Frankhauser, S. (1995). *Valuing Climate Change–The Economics of the Greenhouse*. London: Earthscan.

Gillingham, K. (2012). Selection on anticipated driving and the consumer response to changing gasoline prices.

Griliches, Z. (1957). Hybrid corn: An exploration in the economics of technological change. *Econometrica 25*(4), 501–522.

Haight, R. and W. Smith (1991). Harvesting loblolly pine plantations with hardwood competition and stochastic prices. *Forest science 37*(5), 1266–1282.

Hanak, E. and G. Moreno (2012). California coastal management with a changing climate. *Climatic change 111*(1), 45–73.

Hannah, L., C. Costello, C. Guo, L. Ries, C. Kolstad, D. Panitz, and N. Snider (2012). The impact of climate change on california timberlands. *Climatic change*, 1–15.

Hildebrandt, P. and T. Knoke (2011). Investment decisions under uncertaintya methodological review on forest science studies. *Forest Policy and Economics 13*(1), 1–15.

Holland, J. H. and J. H. Miller (1991). Artificial adaptive agents in economic theory. *The American Economic Review 81*(2), 365–370.

Howarth, R. B., B. M. Haddad, and B. Paton (2000). The economics of energy efficiency: insights from voluntary participation programs. *Energy Policy 28*(6-7), 477 – 486.

Insley, M. (2002). A real options approach to the valuation of a forestry investment. *Journal of Environmental Economics and Management 44*(3), 471–492.

Jaffe, A. B. and R. N. Stavins (1994). The energy-efficiency gap what does it mean? *Energy Policy 22*(10), 804 – 810.

Kelly, D., C. Kolstad, and G. Mitchell (2005). Adjustment costs from environmental change. *Journal of Environmental Economics and Management 50*(3), 468–495.

Linder, M., P. Lasch, and M. Erhard (2000). Alternative forest management strategies under climatic change-prospects for gap model applications in risk analyses. *Silva fennica 34*(2), 101–.

Loewenstein, G. and P. A. Ubel (2008). Hedonic adaptation and the role of decision and experience utility in public policy. *Journal of Public Economics 92*(8-9), 1795 – 1810. Special Issue: Happiness and Public Economics.

Lohmander, P. (1988). Pulse extraction under risk and a numerical forestry application. *Systems Analysis Modelling Simulation 5*(4), 339–354.

Lucas, Robert E., J. (1986). Adaptive behavior and economic theory. *The Journal of Business 59*(4), S401–S426.

Mansur, E. T., R. Mendelsohn, and W. Morrison (2008). Climate change adaptation: A study of fuel choice and consumption in the us energy sector. *Journal of Environmental Economics and Management 55*(2), 175 – 193.

Mendelsohn, R., W. Nordhaus, and D. Shaw (1994). The impact of global warming on agriculture: a ricardian analysis. *The American Economic Review*, 753–771.

Mendelsohn, R., M. Schlesinger, and L. Williams (2000). Comparing impacts across climate models. *Integrated Assessment 1*, 37–48.

Milgrom, P. and I. Segal (2002). Envelope theorem for arbitrary choice sets. *Econometrica 70*(2), 583–601.

Mochon, D., M. I. Norton, and D. Ariely (2008). Getting off the hedonic treadmill, one step at a time: The impact of regular religious practice and exercise on well-being. *Journal of Economic Psychology 29*(5), 632 – 642. Affect, Motivation and Decision Making, Dead Sea conference.

Morck, R., E. Schwartz, and D. Stangeland (1989). The valuation of forestry resources under stochastic prices and inventories. *Journal of Financial and Quantitative Analysis 24*(04), 473–487.

Newman, D. (2002). Forestrys golden rule and the development of the optimal forest rotation literature. *Journal of Forest Economics 8*(1), 5–27.

Nordhaus, W. D. (1994). *Managing the Global Commons: The Economics of Climate Change.* Cambridge, MA: MIT Press.

Plantinga, A. (1998). The optimal timber rotation: An option value approach. *Forest Science 44*(2), 192–202.

Provencher, B. (1995). Structural estimation of the stochastic dynamic decision problems of resource users: An application to the timber harvest decision. *Journal of Environmental Economics and Management 29*(3), 321–338.

Reed, W. and H. Clarke (1990). Harvest decisions and asset valuation for biological resources exhibiting size-dependent stochastic growth. *International Economic Review*, 147–169.

Reeves, L. and R. Haight (2000). Timber harvest scheduling with price uncertainty using markowitz portfolio optimization. *Annals of Operations Research 95*(1), 229–250.

Roberts, G. (2009). Current adaptation measures and policies. *Adaptation of Forest and People to Climate Change–A Global Assessment Report IUFRO World Series Vol. 22.*

Rosenzweig, C. and M. L. Parry (1994). Potential impact of climate change on world food supply. *Nature 367*(6459), 133–.

Rossi-Hansberg, E. and M. L. J. Wright (2007). Establishment size dynamics in the aggregate economy. *The American Economic Review 97*(5), 1639–1666.

Sah, R. and J. Zhao (1998). Some envelope theorems for integer and discrete choice variables. *International Economic Review 39*(3), 623–634.

Sedjo, R. (2010). *Adaptation of forests to climate change: Some estimates.* Resources for the Future.

Smith, B., I. Burton, R. Klein, and J. Wandel (2000). An anatomy of adaptation to climate change and variability. *Climatic change 45*(1), 223–251.

Sohngen, B. and R. Mendelsohn (1998). Valuing the impact of large-scale ecological change in a market: The effect of climate change on U.S. timber. *American Economic Review 88*(4), 686–710.

Sohngen, B., R. Mendelsohn, and R. Sedjo (2001). A global model of climate change impacts on timber markets. *American Journal of Agricultural Economics 26*(2), 326–343.

Spero, J. (2006). California timber yield tax volume and value trends: Historical boe timber harvest data by species, county, ownership.

Spittlehouse, D. L. and R. B. Stewart (2003). Adaptation to climate change in forest management. *BC journal of ecosystems and management 4*(1), 1–.

Stern, N. (2006). Stern review on the economics of climate change.

Thomson, T. (1992). Optimal forest rotation when stumpage prices follow a diffusion process. *Land Economics*, 329–342.

Tol, R. S. (1995). The damage costs of climate change toward more comprehensive calculations. *Environmental and Resource Economics 5*, 353–374.

Tol, R. S. (2002). Estimates of the damage costs of climate change. part 1: Benchmark estimates. *Environmental and Resource Economics 21*, 47–73.

Tol, R. S. (Spring 2009). The economic effects of climate change. *The Journal of Economic Perspectives 23*, 29–51(23).