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Optimal Investment in Energy Efficiency under Uncertainty

Luis M. Abadie[1], José M. Chamorro[2], Mikel Gonzalez-Eguino[3],

This paper deals with the optimal time to invest in an energy efficiency improvement. There is a broad consensus that such investments quickly pay for themselves in lower energy bills and spared emission allowances. However, investments that at first glance seem worthwhile usually are not undertaken. Our aim is to shed some light on this issue. In particular, we try to assess these projects from a financial point of view so as to attract sufficient interest from the investment community. We consider the specific case of a firm or utility already in place that consumes huge amounts of coal and operates under restrictions on carbon dioxide emissions. In order to reduce both coal and carbon costs the firm may undertake an investment to enhance energy efficiency. We consider three sources of uncertainty: the fuel commodity price, the emission allowance price, and the overall investment cost. The parameters of the coal price process and the carbon price process are estimated from observed futures prices. The numerical parameter values are then used in a three-dimensional binomial lattice to assess the value of the option to invest. As usual, maximising this value involves determining the optimal exercise time. Thus we compute the trigger investment cost, i.e., the threshold level below which immediate investment would be optimal. A sensitivity analysis is also undertaken. Our results go some way into explaining the so-called energy efficiency paradox.

Keywords: Energy efficiency, Real options

JEL Classification: Q41, C6

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1 INTRODUCTION

Improvements in energy efficiency (EE) have put a limit to fuel consumption growth in the past (Geller et al. 2006, UNF 2007). Besides they have another basic impact, namely the avoiding of greenhouse gas (GHG) emissions that go hand in hand with fossil fuel combustion (IPCC 2007, IEA 2007). The key to both results is that we do not consume energy as such but energy services (Fouquet 2008); therefore, it can be possible to provide the same amount (level) of energy service with a lower level of energy consumption. Thus, to support governments with their implementation of EE, the IEA recommended the adoption of specific EE policy measures to the G8 summits in 2006, 2007, 2008 and 2009. They cover 25 fields of action across seven priority areas. The IEA estimates that if implemented globally without delay, the proposed actions could save around 8.2 Gt CO2/year by 2030 -equivalent to twice the European Union's current yearly emissions (IEA 2007)-; see Figure 1. Similarly, McKinsey (2007) suggests that the right policies and investments in existing technologies could contribute to a reduction in global energy demand growth by at least half to 2020.

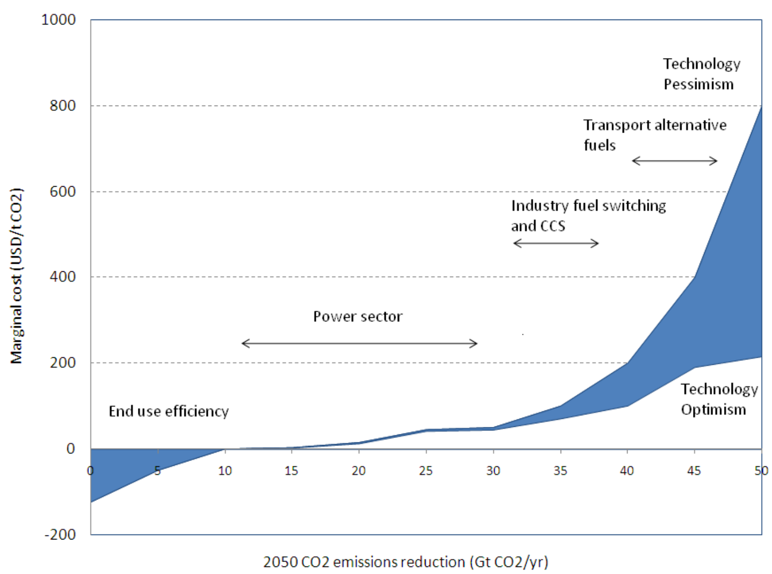

Figure 1: Marginal emission reduction costs for the global energy system, 2050. Source: IEA (2008b).

To the extent that there is a price for carbon dioxide emissions, avoiding them has economic value for firms or utilities that operate in an emissions-constrained environment (or will do so in the future). Of course, this adds to their savings in terms of reduced fuel consumption. And governments must keep

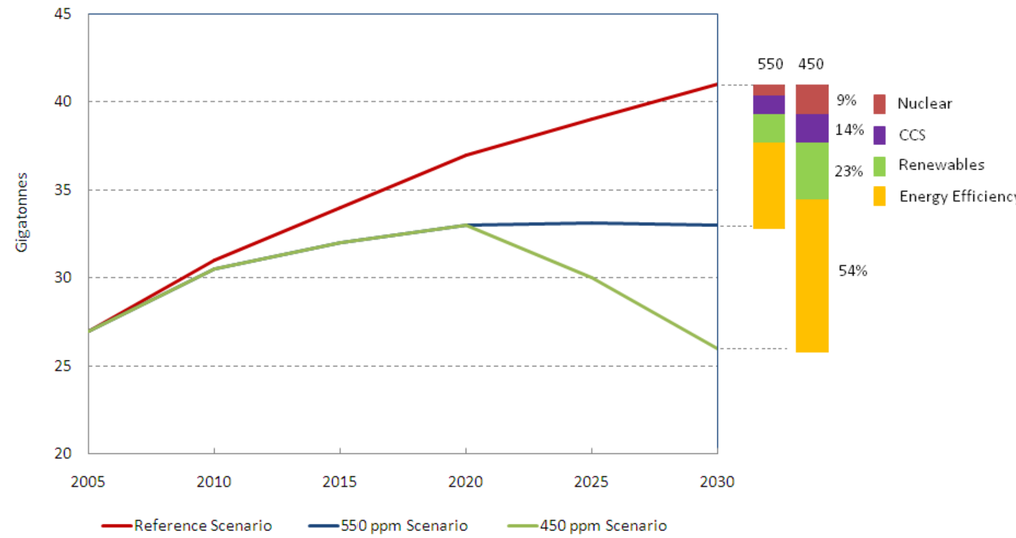

Figure 2: Reductions in energy-related CO_2 emissions in the climate-policy scenarios and desaggregation by technologies. Source: IEA (2008a)

in mind that the benefits of implementing EE extend beyond energy security and climate change mitigation. Experience shows that EE investments can deliver significant co-benefits -including job creation (UNEP 2008) and health improvements (Markandya et al. 2009)-.

There is a broad consensus that such investments quickly pay for themselves in lower energy bills; see Figure 2. As Steven Chu, now the U.S. Secretary of Energy, puts it: "Energy efficiency isn't just low hanging fruit; it's fruit lying on the ground". He has made EE the heart of the Obama Administration's energy strategy. Tighter appliance standards are on a fast track through the Department of Energy bureaucracy. Billions of dollars from the stimulus package are pouring into programs to weatherize and retrofit homes with energy-saving technology.

At the EU level, we can mention early EE policies in the form of legislation covering different activity sectors.[1] More recently the EU has adopted an ambitious policy framework regarding EE in final consumption and other energy services (Directive 2006/32/EC). This piece obliges Member States to set quantitative objectives in terms of energy savings, and measures to promote EE in the provision of energy services. A saving of 9% by the year 2016 was proposed as a reference goal; then each country had to determine the steps required to

[1] Directive on energy efficiency in buildings (2002/91/EC), Directive on the promotion of cogeneration (2004/8/EC), Directive on Eco-design (2005/32/EC).

reach it. The 2008 Climate action and renewable energy package pushes these goals further into the future up to 2020 and beyond;[2] energy consumption must be 20% below the level forecast for that year thanks to enhanced EE in home consumption and also in manufacturing and tertiary sectors.

However, investments that at first glance seem worthwhile usually are not undertaken. For example, around 40% of the potential energy savings from the IEA recommendations, or measures that achieve similar outcomes, remains to be captured. Why? EE continues to face pervasive barriers including insufficient information, principal-agent problems, externality costs that are not reflected in energy prices, and lack of access to capital for energy efficiency investments.[3]

Following Charles (2009), several approaches are being used to address these issues. A first one looks for ways to influence people's energy-using behavior. In this regard some lessons can be learnt from behavioral science. Technology that brings consumers face-to-face with their energy consumptions can also play a role in promoting behavioral change. Another approach aims at fixing "market failures" or overcoming institutional roadblocks. For instance, concerning residential energy use, builders' interests and dwellers' interests typically fall apart when it comes to reducing consumption. In this case, tougher efficiency standards can change that. The Green Paper on energy efficiency (EC 2005) identifies other options to overcome the bottlenecks currently preventing cost-effective efficiencies from being captured; see also EC (2006).[4]

We focus instead on the issue of financing mechanisms for EE. In particular, we feel that the situation traces in part to the challenge of attracting sufficient interest from the investment community. Mills et al.(2006) point out that energy-efficiency experts (as scientists and engineers) and investment decision-makers simply do not speak the same language. Along this line, the Efficiency Valuation Organization (EVO)[5] has launched a set of guidelines to help financial institutions evaluate the risks and quantify the benefits of end-use EE investments. These guidelines are known as the International Energy Efficiency Financing Protocol (IEEFP). They are intended to help EE projects access funding capacity at local financial institutions on commercially attractive terms.[6]

A first barrier to overcome is the traditional 'asset-based' corporate lending approach. Typically it limits lending to 70-80% of the value of the assets financed (or collateral provided). In the particular case of EE projects, there is often little or no collateral value in the equipment once installed; rather, the value is the cash flow generated by the equipment.

A second problem is that many companies that could benefit from EE projects place a low priority on investing capital or using their credit capacity to finance EE. This may be particularly acute in times when corporations

[2] http://ec.europa.eu/environment/climat/climate_action.htm

[3] A comprehensive list of reasons for a lower than expected investment in EE can be found in Linares (2009) and Linares and Lavandeira (2010)

[4] Sáenz de Miera and Muñoz (2009) provide an overview of policy measures aimed to promote EE.

[5] This is a Washington, DC-based non-profit organization.

[6] http://www.evo-world.org/index.php?option=com_content&task=view&id=373&Itemid=373

are cash strapped (as the current scenario). Securing a loan to improve EE, though, would allow to reduce operating costs, thus improving the company's competitiveness and creditworthiness.

Our aim is to further contribute to bridging the gap between the two communities. We note that EE investments lend themselves to financial analysis. In particular, we focus on the valuation of the cash flows that result from investments in EE. We approach them much in the same way as stochastic annuities. We also focus on the timing of the investment, i.e., on the optimal time to invest. Since EE investments are not compulsory, firms can invest immediately but also have the option to wait; and the value of this option can be significant.

Specifically, we analyze investments in EE from the viewpoint of a firm or individual that behaves rationally, i.e., in her best economic interest. The investment or project is valued like a (real) option that is only exercised at the optimal time and is irreversible (the firm cannot disinvest should market conditions turn). The return on this investment is highly uncertain. Uncertainty emanates from energy prices and emission allowance prices, but regulatory uncertainty may come on top of them. We aim to determine the optimal time to invest or, in other words, to learn the conditions under which the investment should be undertaken. We also undertake a sensitivity analysis of our results.

We consider the specific case of a firm or utility already in place that consumes huge amounts of coal and operates under restrictions on carbon dioxide emissions. Obviously the price of both commodities is uncertain. Fortunately, though, both of them are regularly traded on futures markets. This allows to estimate some economic parameters that are relevant for valuation purposes. In order to reduce fuel consumption and carbon emissions the firm or utility may undertake an investment to enhance EE. The cost to such investment, however, is assumed uncertain (either the explicit cost or the intangible cost or both; Dennis 2006). Thus we consider three sources of risk.[7] The parameters of the coal price process and the carbon price process are estimated from futures prices; instead, those of the investment cost are adopted ad hoc. The numerical estimates are then used in a three-dimensional binomial lattice to assess the value of the option to invest. The methodology is similar to that in Boyle et al. 1989. However, our procedure excludes the possibility of negative probabilities, allows for mean-reverting stochastic processes (as opposed to standard geometric Brownian motions), and is later used to value American-type options (as opposed to European-type options). A sensitivity analysis is also undertaken.

Our (base case) results show that the firm will find it optimal to invest in an EE-raising project when the facility has reached about its half useful life. With shorter times to expiration, it is preferable not to invest even if the NPV is positive. This can help to understand the "efficiency gap" and the different perspectives sometimes adopted by engineers and economists when valuing projects. Moreover, as the investment cost increases, longer periods until expiration are required if the option to invest is to be exercised. Some policy issues can be addressed within this framework. Specifically, we briefly

[7] This paper is therefore more general than a previous one by Abadie and Chamorro (2010)

consider the potential effect of a public subsidy to investments that improve EE. We also highlight the impact that uncertainty (or the efforts to reduce it) can have on the optimal time to invest.

The paper is organized as follows. Section 2 sets the theoretical framework. The particular stochastic processes for the three uncertain variables are presented. Since the physical facility where the potential enhancement in EE will take place (and hence the enhancement itself) is finite-lived, we also derive the formula for the value of a stochastic annuity. Section 3 shows the price samples adopted and the estimation procedure that allows to derive numerical values for the underlying parameters. Section 4 explains how the three-dimensional binomial lattice is built. Section 5 comprises the general results and the sensitivity analysis, Section 6 concludes.

2 THE STOCHASTIC MODEL

2.1 Fuel commodity price

We assume that the spot price of a fuel commodity (say, coal) P_t derives from the sum of two components, namely a short-run dynamics S_t (whose expected effect tends to disappear with the passage of time), and a long-run dynamics L_t. The spot price thus equals:[8]

$$P_t = S_t + L_t.$$

The change in the commodity price is given by:[9]

$$dP_t = dS_t + dL_t.$$

We specify the short-term behavior as follows:

$$dS_t = k_S(0 - S_t)dt + \sigma_S S_t dW_t^S = -k_S S_t dt + \sigma_S S_t dW_t^S, \tag{1}$$

where:

k_S: reversion speed. It can be computed as $k_S = \ln 2/t_{1/2}^S$, where $t_{1/2}^S$ denotes the expected time required for the gap between S_t and 0 to halve. We assume a positive speed.

S_t: the current level of the short-run component at time t.

σ_S: the instantaneous volatility of the short-term component, which determines the variance of S_t at t.

dW_t^S: the increment to a standard Wiener process. It is normally distributed with mean zero and variance dt.

[8] The spot price can be unobservable if there is no market for immediate delivery of the commodity.

[9] This setting is similar to that in Schwartz and Smith (2001) but there are some differences. Their model is based on the logarithm of the price; and the long-term equilibrium value follows a non-stationary process. Our model, instead, is based on the price itself; and the long-term value follows a mean-reverting process.

It can easily be shown that the time-t expectation of S_T (with $t \leq T$) is:

$$E(S_T) = S_t e^{-k_S(T-t)}.$$

For $T - t = t^S_{1/2}$ we have $E(S_T) = S_t/2$, and $k_S = \ln 2/t^S_{1/2}$. Also, $E(S_\infty) = 0$ since $k_S \geqq 0$.

The risk-neutral version of the stochastic differential equation for the short-run component is:

$$d\widehat{S}_t = -(k_S + \lambda_S)\widehat{S}_t dt + \sigma_S \widehat{S}_t dW^S_t,$$

where λ_S stands for the corresponding risk premium. The expectation at time t for time T is:

$$E(\widehat{S}_T) = S_t e^{-(k_S+\lambda_S)(T-t)}.$$

In this case $k_S + \lambda_S = \ln 2/t^S_{1/2}$, where $\widehat{t}^S_{1/2}$ is the expected time required (in the risk-neutral world) for the gap between S_t and 0 to halve. Obviously, if $\lambda_S \geq 0$ the gap will narrow earlier. As a consequence, the higher the risk premium the less the influence of S_t on the value of a futures contract, specially for those contracts with longer maturities. Note that the expected value of S_T under risk neutrality coincides with the futures price for delivery at T.

We specify the long-term behavior as a mean-reverting process:

$$dL_t = k_L(L_m - L_t)dt + \sigma_L L_t dW^L_t, \tag{2}$$

where:

L_t: the current level of the long-run component at time t.

L_m: the level to which fuel price tends in the long run (note that the short-term component, as it has been specified, will have no impact in the long term).

k_L: the speed of reversion of the long-run component towards its "normal" level. It can be computed as $k_L = \ln 2/t^L_{1/2}$, where $t^L_{1/2}$ is the expected half-life, i.e. the time required for the gap between L_t and L_m to halve.

σ_L: the instantaneous volatility of the long-term component, which determines the variance of L_t at t.

dW^L_t: the increment to a standard Wiener process. It is normally distributed with mean zero and variance dt.

This specification boils down to $dL_t = \alpha L_t dt + \sigma L_t dW^L_t$ when $L_m = 0$ and $\alpha = -k_L$. Therefore it includes the Geometric Brownian Motion (GBM) as a particular case.

The time-t expectation in the physical world of the long-term component at time T is:

$$E(L_T) = L_m + (L_t - L_m)e^{-k_L(T-t)}.$$

Hence we get:

$$\lim_{k_L \to \infty} E(L_T) = L_m \ ; \lim_{T \to \infty} E(L_T) = L_m \ .$$

For high values of k_L the model provides expected values which are close to L_m; this amounts to the existence of little risk. In this case ($k_L >> 0$), the expected cash flows can be discounted at the risk-free rate r. It can be shown that L_m is actually the expected value of the spot price in the long term:

$$\lim_{T\to\infty} E(P_T) = \lim_{T\to\infty} E(S_T) + \lim_{T\to\infty} E(L_T) = L_m.$$

In order to derive the risk-neutral behavior we subtract the risk premium $\lambda\widehat{L}_t$, which we assume to be proportional to $\widehat{L}_t$.[10] This yields:

$$d\widehat{L}_t = [k_L L_m - (k_L + \lambda_L)\widehat{L}_t]dt + \sigma_L \widehat{L}_t dW_t^L.$$

Therefore, the expected value of the long-term component under risk neutrality is:

$$E_t(\widehat{L}_T) = \frac{k_L L_m}{k_L + \lambda_L} + [L_t - \frac{k_L L_m}{k_L + \lambda_L}]e^{-(k_L+\lambda_L)(T-t)}.$$

The time-t futures price of the commodity for delivery at T, or the time-t risk-neutral expectation is:

$$\begin{aligned} F(S_t, L_t, T, t) &= E_t(\widehat{S}_T) + E_t(\widehat{L}_T) = \\ &= S_t e^{-(k_S+\lambda_S)(T-t)} + \frac{k_L L_m}{k_L + \lambda_L} + [L_t - \frac{k_L L_m}{k_L + \lambda_L}]e^{-(k_L+\lambda_L)(T-t)}. \end{aligned} \tag{3}$$

It comprises two items. The first one is the long-term equilibrium value which the estimated futures curve approaches asymptotically for longer maturities. The second one shows the influence of the gap between the current long-term value and its equilibrium value (this influence also weakens with the passage of time).

Consider the surface that results from futures prices over several days. On each day we have n_i futures prices; we let open the possibility of a growing number of prices as time goes on. Now we show that it is not necessary to know the spot price to compute the parameters on a particular day. Instead, we only need to know the sum of the two components as shown up in futures prices. For maturities τ_1 and τ_2, as seen from time 0 (with $0 < \tau_1 < \tau_2$), we have:

$$\begin{aligned} F(S_t, L_t, \tau_1, t) &= E_t(\widehat{S}_{\tau_1}) + E_t(\widehat{L}_{\tau_1}), \\ F(S_t, L_t, \tau_2, t) &= E_t(\widehat{S}_{\tau_2}) + E_t(\widehat{L}_{\tau_2}). \end{aligned}$$

Since

[10] If the risk premium were specified as a fixed amount, independent of $\widehat{S}_t$, then it would merely be λ. The ensuing formulas would be slightly different.The value of λ can be negative in some instances.

$$\begin{aligned} E_t(\widehat{S}_{\tau_1}) &= S_t e^{-(k_S+\lambda_S)(\tau_1-t)}, \\ E_t(\widehat{S}_{\tau_2}) &= S_t e^{-(k_S+\lambda_S)(\tau_2-t)}, \end{aligned}$$

then:

$$E_t(\widehat{S}_{\tau_2}) = E_t(\widehat{S}_{\tau_1}) e^{-(k_S+\lambda_S)(\tau_2-\tau_1)}.$$

This expression allows the usage of maturity gaps $\tau_2 - \tau_1$ which can be constant between futures contracts. Thus, $\tau_2 - \tau_1 = 1/12$ for futures contracts with monthly maturities that are uniformly separated between them.

Regarding the long-term component:

$$\begin{aligned} E_t(\widehat{L}_{\tau_1}) &= \frac{k_L L_m}{k_L + \lambda_L} + [L_t - \frac{k_L L_m}{k_L + \lambda_L}] e^{-(k_L+\lambda_L)(\tau_1-t)}, \\ E_t(\widehat{L}_{\tau_2}) &= \frac{k_L L_m}{k_L + \lambda_L} + [L_t - \frac{k_L L_m}{k_L + \lambda_L}] e^{-(k_L+\lambda_L)(\tau_2-t)}. \end{aligned}$$

Hence we get:

$$E_t(\widehat{L}_{\tau_2}) = \frac{k_L L_m}{k_L + \lambda_L} + [E_t(\widehat{L}_{\tau_1}) - \frac{k_L L_m}{k_L + \lambda_L}] e^{-(k_L+\lambda_L)(\tau_2-\tau_1)}.$$

Therefore:

$$\begin{aligned} F(E_t(\widehat{S}_{\tau_1}), E_t(\widehat{L}_{\tau_1}), \tau_2, \tau_1) &= E_t(\widehat{S}_{\tau_1}) e^{-(k_S+\lambda_S)(\tau_2-\tau_1)} + \frac{k_L L_m}{k_L + \lambda_L} + \\ &\quad + [E_t(\widehat{L}_{\tau_1}) - \frac{k_L L_m}{k_L + \lambda_L}] e^{-(k_L+\lambda_L)(\tau_2-\tau_1)}. \end{aligned}$$

Taking τ_1 as the maturity of the nearest contract, since

$$E_t(\widehat{S}_{\tau_1}) = F(S_t, L_t, \tau_1, t) - E_t(\widehat{L}_{\tau_1}), \tag{4}$$

in the end we have:

$$\begin{aligned} F(E_t(\widehat{S}_{\tau_1}), E_t(\widehat{L}_{\tau_1}), \tau_2, \tau_1) &= \left[F(S_t, L_t, \tau_1, t) - E_t(\widehat{L}_{\tau_1})\right] e^{-(k_S+\lambda_S)(\tau_2-\tau_1)} + \\ &\quad + \frac{k_L L_m}{k_L + \lambda_L} + \left[E_t(\widehat{L}_{\tau_1}) - \frac{k_L L_m}{k_L + \lambda_L}\right] e^{-(k_L+\lambda_L)(\tau_2-\tau_1)}. \end{aligned} \tag{5}$$

With this formula we can estimate $k_S + \lambda_S$, $k_L + \lambda_L$, $\frac{k_L L_m}{k_L+\lambda_L}$ and $E_t(\widehat{L}_{\tau_1})$ from futures prices on a given day or a set of days. Hence we can also compute the initial value of the short-term component $E_t(\widehat{S}_{\tau_1})$; see equation (4).[11]

[11] From these estimates we cannot derive the value of some isolated parameters such as $k_S, \lambda_S, k_L, \lambda_L$ and L_m. To that end we should resort to the behavior of the spot price (assuming it is available) or futures prices in the physical world as time t evolves. Anyway we do not need the above parameters for valuation purposes.

2.2 Emission allowance price

We adopt a standard GBM process for carbon price:

$$dC_t = \alpha_C C_t dt + \sigma_C C_t dW_t^C, \tag{6}$$

where C_t denotes the price of the emission allowance at time t. The instantaneous drift rate is denoted by α_C, while σ_C stands for the instantaneous volatility of carbon price changes. In a risk-neutral setting the stochastic differential equation is:

$$d\widehat{C}_t = (\alpha_C - \lambda_C)\widehat{C}_t dt + \sigma_C \widehat{C}_t dW_t^C. \tag{7}$$

λ_C stands for the premium related to carbon price risk. The expression for the futures price is a particular case of that used for the fuel commodity, specifically:

$$F(C_t, T, t) = E_t(\widehat{C}_T) = C_t e^{(\alpha_C - \lambda_C)(T-t)}. \tag{8}$$

Market data from futures contracts on CO_2 emission allowances show that carbon price rises progressively between successive futures contracts, for example those with December maturities. This is consistent with our choice of a GBM process.

2.3 Overall investment cost

Again we assume a GBM model. Denoting I_t as the investment outlay [12] at time t, it evolves stochastically according to:

$$dI_t = \alpha_I I_t dt + \sigma_I I_t dW_t^I. \tag{9}$$

The risk-neutral version is:

$$d\widehat{I}_t = (\alpha_I - \lambda_I)\widehat{I}_t dt + \sigma_I \widehat{I}_t dW_t^I. \tag{10}$$

λ_I stands for the premium related to the risk concerning the amount to disburse. The expression for the futures price is a particular case of that used for the fuel commodity, specifically:

$$F(I_t, T, t) = E_t(\widehat{I}_T) = I_t e^{(\alpha_I - \lambda_I)(T-t)}. \tag{11}$$

If the short-term dynamics of the spot price were not relevant for an investment to increase energy efficiency, we would only need to calibrate the long-term dynamics. In that case we should determine three correlation coefficients:

$$dW_t^L dW_t^C = \rho_{LC} dt \; ; \; dW_t^L dW_t^I = \rho_{LI} dt \; ; \; dW_t^C dW_t^I = \rho_{CI} dt. \tag{12}$$

[12] The term I_t refers to the time-t present value of all the investment costs (whether they are disbursed all at once or sequentially over time, be they tangible or intangible, and net of whatever public subsidies received).

2.4 Value of a stochastic annuity between times τ_1 and τ_2

Assume we are deciding whether to invest or not at a given time. Therefore we need to know the present value of the prospective profits accruing to the investment, V. We deal with a stochastic income from each unit of fuel saved and each emission allowance spared. The value of this income can be computed as follows:

$$V = \int_{\tau_1}^{\tau_2} e^{-rt} F(S_t, L_t, T, t) + Q \int_{\tau_1}^{\tau_2} e^{-rt} F(C_t, T, t), \tag{13}$$

where Q stands for the tons of carbon dioxide avoided per unit of fuel saved.[13] The most expensive, dirtiest fuels would be the natural candidates for investments that enhance energy efficiency,[14] provided they are technically feasible.

The value of the annuity emanates from three sources (see equations (5) and (8)):

$$V(S_0, L_0, C_0) = V_1(S_0) + V_2(L_0) + V_3(C_0). \tag{14}$$

a) The effect of the short-run fuel price:

$$V_1(S_0) = \frac{S_0}{k_S + \lambda_S + r}[e^{-(k_S+\lambda_S+r)\tau_1} - e^{-(k_S+\lambda_S+r)\tau_2}]. \tag{15}$$

When the maturity τ_1 is rather distant from zero and the sum $(k_S + \lambda_S + r)$ is high this component can be negligible.

b) The effect of the long-run fuel price:

$$V_2(L_0) = \frac{k_L L_m}{r(k_L + \lambda_L)}[e^{-r\tau_1} - e^{-r\tau_2}] + [L_0 - \frac{k_L L_m}{k_L + \lambda_L}]\frac{[e^{-(k_L+\lambda_L+r)\tau_1} - e^{-(k_L+\lambda_L+r)\tau_2}]}{k_L + \lambda_L + r}. \tag{16}$$

c) The effect of the emission allowances spared:

$$V_3(C_0) = Q\frac{C_0}{\lambda_C + r - \alpha}[e^{-(\lambda_C+r-\alpha_C)\tau_1} - e^{-(\lambda_C+r-\alpha_C)\tau_2}]. \tag{17}$$

Note that this valuation only requires knowledge of the parameters derived from futures prices, i.e., those expected to prevail in a risk-neutral world. At a time when the stochastic variables take on the values (S_0, L_0, C_0), starting from the estimates of $k_S + \lambda_S$, $k_L + \lambda_L$, $\frac{k_L L_m}{k_L+\lambda_L}$, and $\alpha_C - \lambda_C$ we could immediately compute the value of an annuity between dates τ_1 and τ_2.

The Net Present Value at the initial time is computed as:

$$NPV_0 = V(S_0, L_0, C_0) - I_0.$$

[13] Previously, if coal prices and carbon prices are quoted in different monetary units, the exchange rate will be used to convert them.

[14] Typically the cheapest fuels turn out to be the dirtiest ones.

Table 1. Summary statistics for Appalachian coal futures (NYMEX). Daily data from 04/01/07 to 03/06/09			
	Observations	Avg. Price ($/ton)	Std. Dev.
All contracts	22,536	68.22	21.74
1. Closest	607	68.22	26.99
2. Closest	607	65.50	24.68
3. Closest	607	66.50	22.96
4. Closest	607	67.46	21.22
5. Closest	607	68.15	20.02
6. Closest	607	68.33	19.66
7. Closest	607	68.39	19.08
8. Closest	446	71.78	19.72
9. Closest	224	78.13	22.89
Note: The periods beteween categories is four months.			

Similarly, at a given time t when we observe (S_t, L_t, C_t) and I_t we compute:

$$NPV_t = V(S_t, L_t, C_t) - I_t.$$

If the short-run component has a limited effect, which happens when $k_S + \lambda_S + r >> 0$ and $\tau_1 >> 0$, a good approximation is:

$$NPV_t = V_2(L_t) + V_3(C_t) - I_t.$$

3 DATA AND CALIBRATION

3.1 Data

The sample consists of daily futures prices of coal on the NYMEX from 04/01/2007 to 03/06/2009, or 607 days. Each day there is a variable number of futures prices, depending on their maturity. The minimum number of contracts on a day is 30, whereas the maximum is 41, which takes place at the final part of the sample.

With regard to the futures market for EU emission allowances, we use the same trading days as for coal futures. In this case, though, carbon prices are taken from the European Climate Exchange (ECX); the specific contract is referred to as EUA Futures.

The last part of the series includes contracts with maturity December-2013 and December-2014. These contracts thus fall beyond the Kyoto Protocol's expiration.

As an example, Figures 6-7-8-9 show the market in different daily situations such as backwardation, contango, or mixed cases. The curve shown results from the sum of two components, namely the short- and long-term components. Though in some cases the curve is U-shaped or inverted-U-shaped, the first part

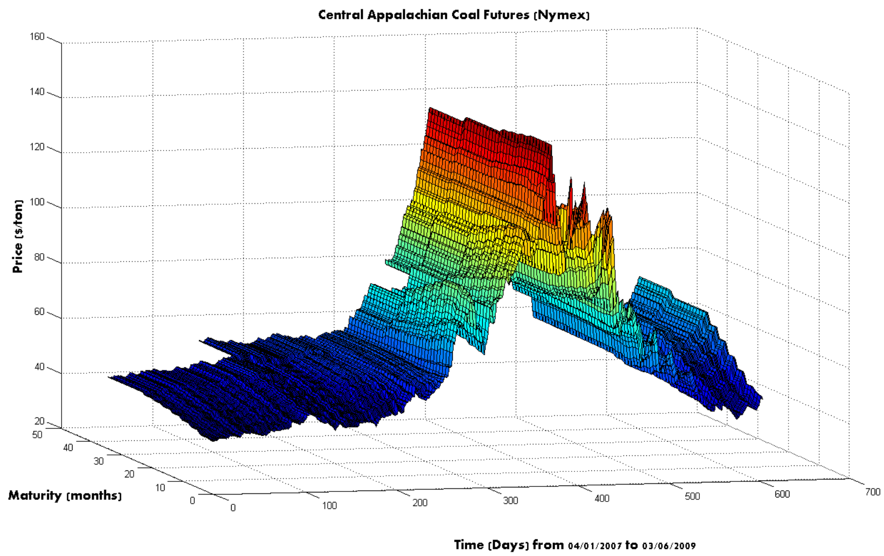

Figure 3: Central Appalachian Coal Futures (NYMEX) over thirty months.

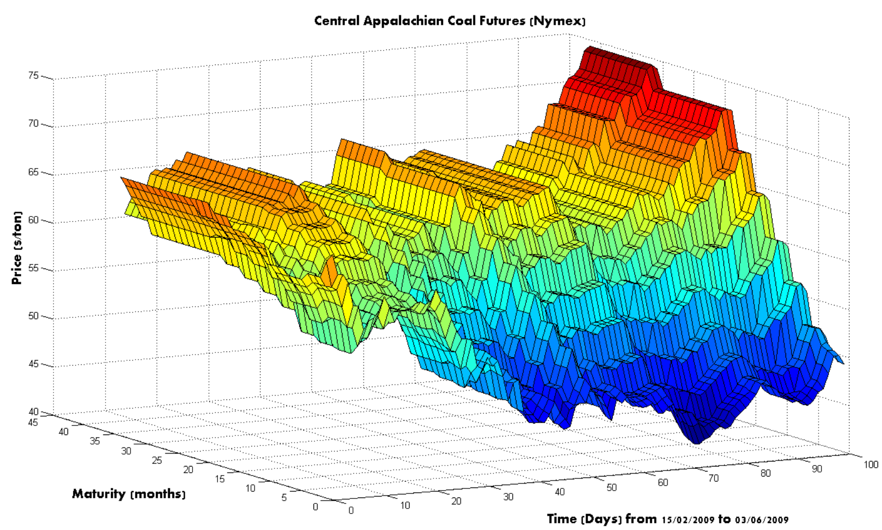

Figure 4: Central Appalachian Coal Futures (NYMEX) over last four months.

Table 2. Summary statistics for EU allowance futures (ECX). Daily data from 01/02/09 to 09/23/09			
	Observations	Avg. Price (€/tonne)	Std. Dev.
All contracts	1,116	18.33	3.48
Dec 2009	186	16.43	2.67
Dec 2010	186	17.05	2.72
Dec 2011	186	17.86	2.80
Dec 2012	186	19.01	2.97
Dec 2013	186	20.59	2.96
Dec 2014	186	22.02	3.10

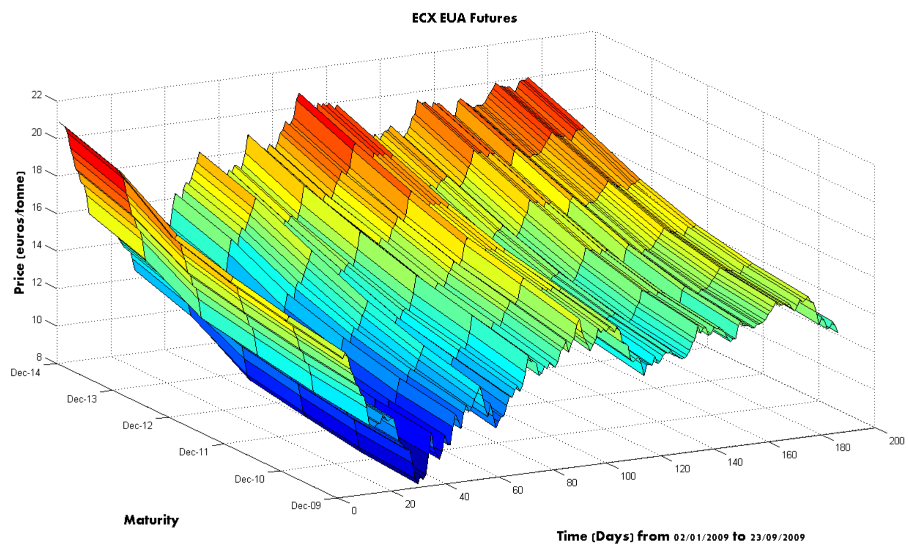

Figure 5: Futures contracts on EU allowances (ECX) over nine months.

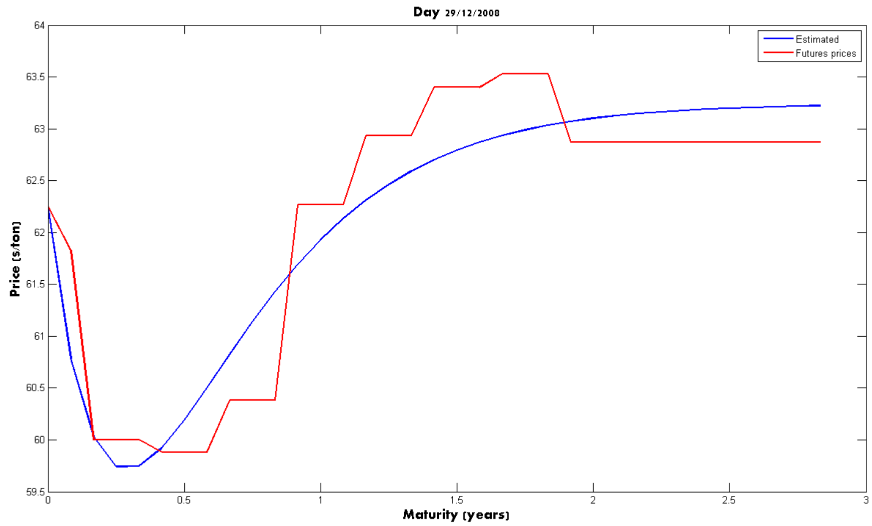

Figure 6: Mixed case: backwardation followed by contango.

of the curve disappears beyond maturities of 0.5 years; in other words, only the long-term component remains.

Analysis of the series suggests that the short-term component has a very limited effect on the annuity value; its impact on the futures curve disappears after a few months. Instead, many investments to increase energy efficiency take some time between decision and first results (savings). If this time lapse stretches over a few months then the short-run dynamics has a negligible effect, the more so in investments whose profits will arise over many years.

3.2 Calibration

3.2.1 Parameters in the coal price process

We estimate the parameters of the coal price process considering only the long-term dynamics. We use the futures prices over 50 days ranging from 03/24/09 to 06/03/09. These days are the last days in our sample. If we took earlier dates, we would get into the price-bubble period on the commodities markets.

If we undertake the valuation of future physical flows of commodities at a given time t, the receipt of which is absolutely certain, valuation should rest on the time-t futures curve. Therefore, our model must leave room for $k_L + \lambda_L$ and $\frac{k_L L_m}{k_L+\lambda_L}$ to change in value on a daily basis. Hence we accept that some items such as λ_L or λ_C can vary over time. Thus we recognize that the risk premium changes as is the case in financial markets and commodity markets. Despite the

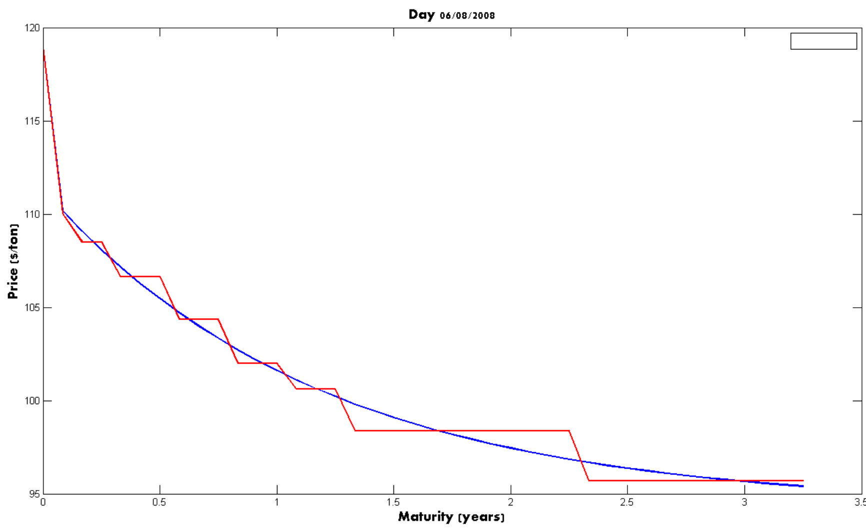

Figure 7: Backwardation all along.

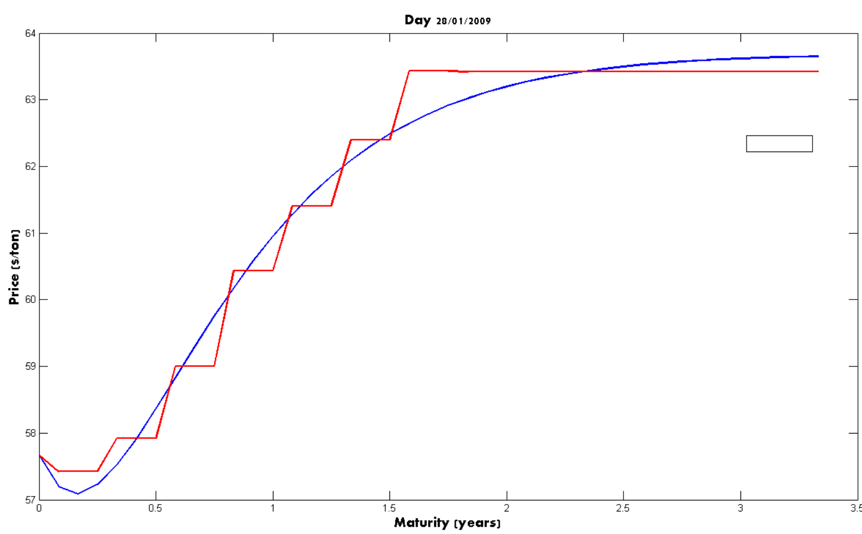

Figure 8: Contango for the most part.

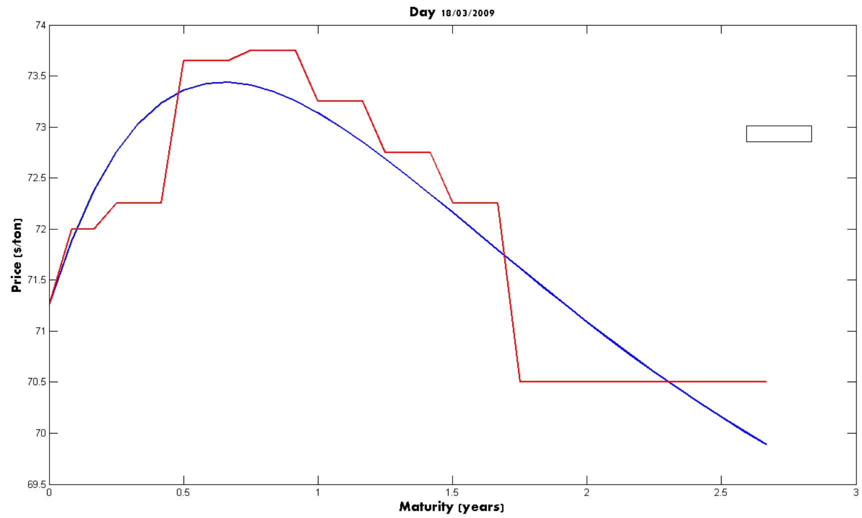

Figure 9: Mixed case: contango followed by backwardation.

variability of $k_L + \lambda_L$ and $\frac{k_L L_m}{k_L + \lambda_L}$ with time, we are going to estimate an average value that best fits the series of daily values.[15] We further use these average values as an estimate of future behavior.

The calibration process consists of two steps. It has some similarities with the process followed by Cortazar and Schwartz (2001). In the first step, using the prices on each day and non-linear least-squares, we derive the curve that best fits the prices on that day, which provides an estimate of the parameters in expression (5). As already mentioned, we do not need a spot price (which sometimes does not exist). This estimation of the parameters refers to price behavior under risk neutrality.

Our process has several advantages:

a) It allows direct usage of futures prices without any need of an unobservable variable, as the spot price P_t (which is used when the Kalman filter is adopted).

b) The time lapses between prices are constant. There is no initial term to maturity of varying length, which usually is given by the time between the spot price and the nearest futures price.

c) It is possible, without complicating estimation and contributing to it, to use all the futures prices available on a given day. This is not typically the case in Kalman filter-based estimations, where a limited number of futures prices are chosen.

[15] Somteching similar happens with volatility. Though it changes from one day to the next, usually it suffices to estimate one single value when trying to value long-term cash flows. This would not be the case if we were trying to model only the behavior of the spot price.

d) It allows to use a variable number of futures prices over time. This is convenient since new contracts with longer maturities are introduced periodically, the prices of which can be of interest for long-term valuations.

e) It is possible to compute confidence intervals for the estimations of each day. The same holds for the estimates of $k_L + \lambda_L$, and $\frac{k_L L_m}{k_L+\lambda_L}$ computed as the average of the daily estimates. These daily values are derived in the second step.

Upon the calibration on each of the 50 days, we compute the corresponding average value. They are shown in Table 3.

Table 3. Average value of the coal parameters.			
Parameter	Estimate	Std. error	t-ratio
$\frac{k_L L_m}{k_L+\lambda_L}$	70.13	0.6503	107.8
$k_L + \lambda_L$	0.62	0.0103	60.05

The most relevant parameter for long-term valuations is $\frac{k_L L_m}{k_L+\lambda_L}$ because of the high value of $(k_L + \lambda_L)$. Figures 10-11 display the values on each day and the 95-percent confidence intervals.

Regarding estimation of the volatilities, first we estimate the series of the current long-term component $E_t(L_t)$ from the nearest futures contract $F(S_t, L_t, \tau_1, t)$ using the parameters estimated for each day:

$$E_t(\widehat{L}_t) = \left[F(S_t, L_t, \tau_1, t) - \frac{k_L L_m}{k_L + \lambda_L}\right] e^{(k_L+\lambda_L)(\tau_1 - t)} + \frac{k_L L_m}{k_L + \lambda_L}.$$

The resulting values of $E_t(\widehat{L}_t)$ from a regression,[16] using the expression for the behavior in the physical world, allow to compute a volatility of $\widehat{\sigma}_L = 0.2850$.

3.2.2 Parameters in the allowance price process

The model is calibrated with daily futures prices from 01/02/2009 to 09/23/2009. Previously, prices from the ECX, which are measured in €/tonne, are converted to \$/ton.[17] Calibration proceeds along the same steps as before. We estimate $(\alpha_C - \lambda_C)$ for each day by non-linear least squares. The result appears in Table 4. Volatility is derived by similar procedures. We get $\widehat{\sigma}_C = 0.5622$. Residuals from the regression allow to compute the correlation coefficient $\widehat{\rho}_{LC} = 0.0525$.

Table 4. Average value of the carbon parameters			
Parameter	Estimate	Std. error	t-ratio
$\alpha_C - \lambda_C$	0.056	0.0004	115.8

[16] The regression is based on equation (2).

[17] 1 tonne= 1.10231136 tons. The exchange rate is taken from the Bank of Spain's fixing rate. This conversion does not affect the estimate of the slope. And the change in the estimate of the volatility is very small: from $\widehat{\sigma}_E = 0.5254$ (in €/tonne) to $\widehat{\sigma}_E = 0.5622$.

Figure 10: Risk-neutral speed of reversion $(k_L + \lambda_L)$ over time and confidence interval.

The expected growth rate of carbon prices under risk neutrality, $(\alpha_C - \lambda_C)$, along with the volatility σ_C, are fundamental components to the valuation process. In view of Figure 12, our estimate $\alpha_C - \lambda_C = 0.056$ can be considered a reasonable value.

Another key ingredient to valuation is the amount of carbon dioxide that is avoided for each ton of coal that ceases to be consumed. Obviously this depends on coal quality. Using data from EIA (2006), we can compute the emission factors as shown in Table 5.[18]

Table 5. Emission factors from different coal types.

Coal type	tons CO_2 / ton coal
Lignite	1.3958
Subbituminous	1.8580
Bituminous	2.4657
Anthracite	2.8425

In this paper we assume bituminous coal as the input fuel, hence $Q = 2.4657$.

[18] One ton of Anthracite pollutes more than one tone of Bituminous coal. But a given amount of power will take less Anthracite than Bituminous. Sometimes, though, there is no choice because the type of coal fired can be determined by the physical proximity of coal mines.

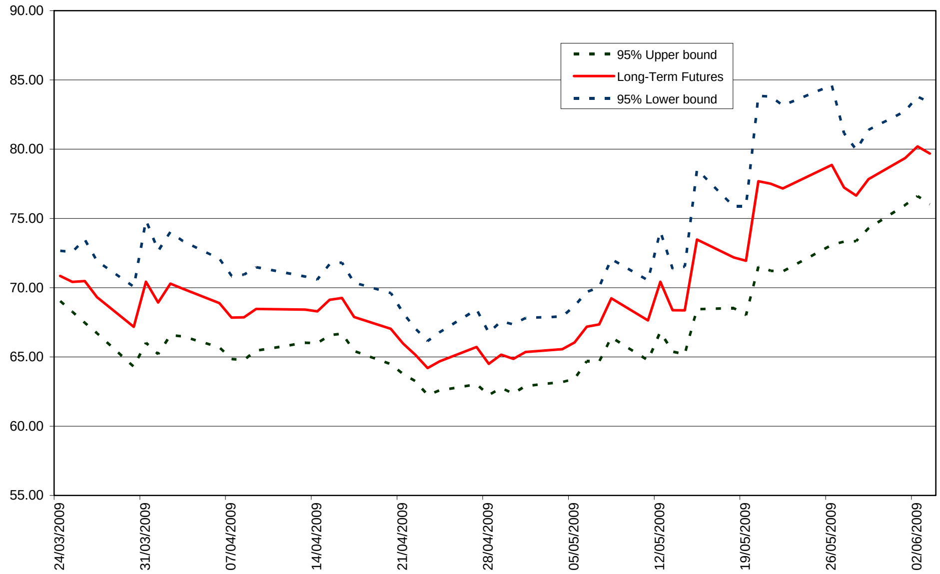

Figure 11: Long-term component $k_L L_m/(k_L + \lambda_L)$ over time and confidence interval.

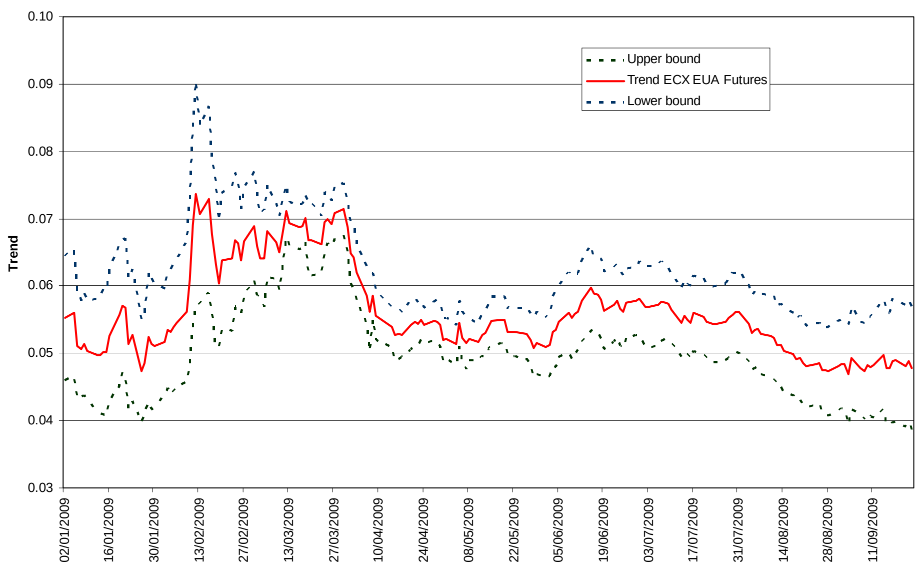

Figure 12: Risk-neutral drift rate $(\alpha_C - \lambda_C)$ over time and confidence interval.

4 THE THREE-DIMENSIONAL LATTICE

4.1 Building the lattice

First we take natural logarithms of the prices:

$$x_I \equiv \ln \widehat{I}_t;\ x_L \equiv \ln \widehat{L}_t;\ x_C \equiv \ln \widehat{C}_t.$$

Applying Ito's Lemma, for the dynamics of the investment cost we have:

$$dx_I = (\alpha_I - \lambda_I - \frac{1}{2}\sigma_I^2)dt + \sigma_I dW_t^I = \nu_I dt + \sigma_I dW_t^I.$$

For the long-run dynamics of coal price we have:

$$dx_L = \left[\frac{k_L(L_m - \widehat{L}_t)}{\widehat{L}_t} - \lambda_L - \frac{1}{2}\sigma_L^2\right] dt + \sigma_L dW_t^L = \nu_L dt + \sigma_L dW_t^L, \qquad (18)$$

which can be rewritten as:

$$dx_L = \left[\frac{1}{\widehat{L}_t}\frac{k_L L_m}{k_L + \lambda_L}(k_L + \lambda_L) - (k_L + \lambda_L) - \frac{1}{2}\sigma_L^2\right] dt + \sigma_L dW_t^L = \nu_L dt + \sigma_L dW_t^L.$$

For the dynamics of the allowance price we have:

$$dx_C = (\alpha_C - \lambda_C - \frac{1}{2}\sigma_C^2)dt + \sigma_C dW_t^C = \nu_C dt + \sigma_C dW_t^C. \qquad (19)$$

Note that, except for volatilities, all the parameters required for using the above formulas can be estimated in the risk-neutral world from futures prices.

With three dimensions in each node of the lattice, it is possible to move to $2^3 = 8$ different states of nature. Thus there are eight probabilities to be computed, in addition to three incremental values (Δx_I; Δx_L; Δx_C). For this purpose we have ten equations.

The first equation establishes that the probabilities must sum to one:

$$p_{uuu} + p_{uud} + p_{udu} + p_{udd} + p_{duu} + p_{dud} + p_{ddu} + p_{ddd} = 1.$$

The next three impose the conditions for consistency regarding the second moment:

$$\begin{aligned} E(\Delta x_I^2) &= (p_{uuu} + p_{uud} + p_{udu} + p_{udd})\Delta x_I^2 + (p_{duu} + p_{dud} + p_{ddu} + p_{ddd})\Delta x_I^2 = \\ &= \sigma_I^2 \Delta t + \nu_I^2 (\Delta t)^2 \simeq \sigma_I^2 \Delta t, \end{aligned}$$

$$E(\Delta x_L^2) = \Delta x_L^2 = \sigma_L^2 \Delta t + \nu_L^2 (\Delta t)^2 \simeq \sigma_L^2 \Delta t,$$

$$E(\Delta x_C^2) = \Delta x_C^2 = \sigma_C^2 \Delta t + \nu_C^2 (\Delta t)^2 \simeq \sigma_C^2 \Delta t.$$

When the increments Δt in the lattice are small, the term $(\Delta t)^2 \simeq 0$. These equations allow to directly compute the increments:

$$\Delta x_I = \sigma_I \sqrt{\Delta t};\ \Delta x_L = \sigma_L \sqrt{\Delta t};\ \Delta x_C = \sigma_C \sqrt{\Delta t}.$$

The next three equations require the probabilities to be consistent with observed correlations:

$$\begin{aligned} E(\Delta x_I \Delta x_L) &= (p_{uuu} + p_{uud} - p_{udu} - p_{udd} - p_{duu} - p_{dud} + p_{ddu} + p_{ddd})\Delta x_I \Delta x_L = \\ &= \rho_{IL}\sigma_I\sigma_L\Delta t + \nu_I\nu_L(\Delta t)^2, \end{aligned}$$

$$\begin{aligned} E(\Delta x_I \Delta x_C) &= (p_{uuu} - p_{uud} + p_{udu} - p_{udd} - p_{duu} + p_{dud} - p_{ddu} + p_{ddd})\Delta x_I \Delta x_C = \\ &= \rho_{IC}\sigma_I\sigma_C\Delta t + \nu_I\nu_C(\Delta t)^2, \end{aligned}$$

$$\begin{aligned} E(\Delta x_L \Delta x_C) &= (p_{uuu} - p_{uud} - p_{udu} + p_{udd} + p_{duu} - p_{dud} - p_{ddu} + p_{ddd})\Delta x_L \Delta x_C = \\ &= \rho_{LC}\sigma_L\sigma_C\Delta t + \nu_L\nu_C(\Delta t)^2. \end{aligned}$$

Remembering that $(\Delta t)^2 \simeq 0$ and the values for Δx_I, Δx_L, and Δx_C, we get:

$$p_{uuu} + p_{uud} - p_{udu} - p_{udd} - p_{duu} - p_{dud} + p_{ddu} + p_{ddd} = \rho_{IL}$$

$$p_{uuu} - p_{uud} + p_{udu} - p_{udd} - p_{duu} + p_{dud} - p_{ddu} + p_{ddd} = \rho_{IC}$$

$$p_{uuu} - p_{uud} - p_{udu} + p_{udd} + p_{duu} - p_{dud} - p_{ddu} + p_{ddd} = \rho_{LC}$$

The last three equations establish the conditions for consistency with the first moment:

$$E(\Delta x_I) = (p_{uuu} + p_{uud} + p_{udu} + p_{udd} - p_{duu} - p_{dud} - p_{ddu} - p_{ddd})\Delta x_I = \nu_I \Delta t,$$

$$E(\Delta x_L) = (p_{uuu} + p_{uud} - p_{udu} - p_{udd} + p_{duu} + p_{dud} - p_{ddu} - p_{ddd})\Delta x_L = \nu_L \Delta t,$$

$$E(\Delta x_C) = (p_{uuu} - p_{uud} + p_{udu} - p_{udd} + p_{duu} - p_{dud} + p_{ddu} - p_{ddd})\Delta x_C = \nu_C \Delta t.$$

From them we derive:

$$p_{uuu} + p_{uud} + p_{udu} + p_{udd} - p_{duu} - p_{dud} - p_{ddu} - p_{ddd} = \frac{\nu_I \sqrt{\Delta t}}{\sigma_I},$$

$$p_{uuu} + p_{uud} - p_{udu} - p_{udd} + p_{duu} + p_{dud} - p_{ddu} - p_{ddd} = \frac{\nu_L \sqrt{\Delta t}}{\sigma_L},$$

$$p_{uuu} - p_{uud} + p_{udu} - p_{udd} + p_{duu} - p_{dud} + p_{ddu} - p_{ddd} = \frac{\nu_C \sqrt{\Delta t}}{\sigma_C}.$$

We thus have seven equations and eight unknowns. In principle, several solutions are possible. However, we adopt the method suggested by Boyle et al. (2001). This way we get the following probabilities, which satisfy the above equations:

$$p_{uuu} = \frac{1}{8}[1 + \rho_{IL} + \rho_{IC} + \rho_{LC} + \sqrt{\Delta t}(\frac{\nu_I}{\sigma_I} + \frac{\nu_L}{\sigma_L} + \frac{\nu_C}{\sigma_C})],$$

$$p_{uud} = \frac{1}{8}[1 + \rho_{IL} - \rho_{IC} - \rho_{LC} + \sqrt{\Delta t}(\frac{\nu_I}{\sigma_I} + \frac{\nu_L}{\sigma_L} - \frac{\nu_C}{\sigma_C})],$$

$$p_{udu} = \frac{1}{8}[1 - \rho_{IL} + \rho_{IC} - \rho_{LC} + \sqrt{\Delta t}(\frac{\nu_I}{\sigma_I} - \frac{\nu_L}{\sigma_L} + \frac{\nu_C}{\sigma_C})],$$

$$p_{udd} = \frac{1}{8}[1 - \rho_{IL} - \rho_{IC} + \rho_{LC} + \sqrt{\Delta t}(\frac{\nu_I}{\sigma_I} - \frac{\nu_L}{\sigma_L} - \frac{\nu_C}{\sigma_C})],$$

$$p_{duu} = \frac{1}{8}[1 - \rho_{IL} - \rho_{IC} + \rho_{LC} + \sqrt{\Delta t}(-\frac{\nu_I}{\sigma_I} + \frac{\nu_L}{\sigma_L} + \frac{\nu_C}{\sigma_C})],$$

$$p_{dud} = \frac{1}{8}[1 - \rho_{IL} + \rho_{IC} - \rho_{LC} + \sqrt{\Delta t}(-\frac{\nu_I}{\sigma_I} + \frac{\nu_L}{\sigma_L} - \frac{\nu_C}{\sigma_C})],$$

$$p_{ddu} = \frac{1}{8}[1 + \rho_{IL} - \rho_{IC} - \rho_{LC} + \sqrt{\Delta t}(-\frac{\nu_I}{\sigma_I} - \frac{\nu_L}{\sigma_L} + \frac{\nu_C}{\sigma_C})],$$

$$p_{ddd} = \frac{1}{8}[1 + \rho_{IL} + \rho_{IC} + \rho_{LC} + \sqrt{\Delta t}(-\frac{\nu_I}{\sigma_I} - \frac{\nu_L}{\sigma_L} - \frac{\nu_C}{\sigma_C})].$$

These probabilities have the same structure as those derived by Boyle et al. 1989; the terms ν_I, ν_L, ν_C, though, are different. Our development allows for mean-reverting stochastic processes, and is later used to value American-type options (unlike Boyle et al. who value European-type options).

Negative probabilities cannot be accepted. To avoid this possibility we apply Bayes's Rule which decomposes the former probabilities into a product of conditional and marginal probabilities. We adopt a procedure which is similar to that in Bastian-Pinto et al. 2009. However, we consider three sources of risk (instead of two).

The conditional probabilities for x_I are:

$$p_u = p_{uuu} + p_{uud} + p_{udu} + p_{udd} = \frac{1}{2} + \frac{1}{2}\sqrt{\Delta t}\frac{\nu_I}{\sigma_I},$$

$$p_d = p_{duu} + p_{dud} + p_{ddu} + p_{ddd} = \frac{1}{2} - \frac{1}{2}\sqrt{\Delta t}\frac{\nu_I}{\sigma_I}.$$

It must be $p_u + p_d = 1$, with neither of them greater than one and less than zero. Therefore some nodes are censured as follows:

$$p_u^* = \max(0, \min(1, p_u)) \ ; \ p_d^* = 1 - p_u^*.$$

Now we derive the conditional probabilities of x_L in the following way:

$$p_{u/u} = \frac{p_{uuu} + p_{uud}}{p_u},$$

$$p_{d/u} = \frac{p_{udu} + p_{udd}}{p_u}.$$

These probabilities only make sense if $p_u^* > 0$, in which case it must be $p_{u/u} + p_{d/u} = 1$. Besides, they must be both between zero and one. If this does not hold at some node, we censure them as follows:

$$\begin{aligned} if \ p_u^* &> 0 \text{ then } p_{u/u}^* = \max(0, \min(1, p_{u/u})) \ ; \ p_{d/u}^* = 1 - p_{u/u}^*, \\ if \ p_u^* &= 0 \text{ then } p_{u/u}^* = 0 \ ; \ p_{d/u}^* = 0. \end{aligned}$$

We similarly arrive at:

$$\begin{aligned} if \ p_d^* &> 0 \text{ then } \ p_{u/d}^* = \max(0, \min(1, p_{u/d})) \ ; \ p_{d/d}^* = 1 - p_{u/d}^*, \\ if \ p_d^* &= 0 \text{ then } p_{u/d}^* = 0 \ ; \ p_{d/d}^* = 0. \end{aligned}$$

In case $p_{u/u}^* > 0$ the conditional probabilities of x_E are derived as:

$$p_{u/u/u} = \frac{p_{uuu}}{p_{uuu} + p_{uud}},$$

$$p_{d/u/u} = \frac{p_{duu}}{p_{uuu} + p_{uud}}.$$

Thus we get:

$$p_{u/u/u}^* = \max(0, \min(1, p_{u/u/u})) \ ; \ p_{d/u/u}^* = 1 - p_{u/u/u}^*.$$

Analogously:

if $p_{d/u}^* > 0$:

$$p^*_{u/d/u} = \max(0, \min(1, p_{u/d/u}))\ ;\ p^*_{d/d/u} = 1 - p^*_{u/d/u};$$

if $p^*_{u/d} > 0$:

$$p^*_{u/u/d} = \max(0, \min(1, p_{u/u/d}))\ ;\ p^*_{d/u/d} = 1 - p^*_{u/u/d};$$

if $p^*_{d/d} > 0$:

$$p^*_{u/d/d} = \max(0, \min(1, p_{u/d/d}))\ ;\ p^*_{d/d/d} = 1 - p^*_{u/d/d}.$$

In the end, the new probabilities are simply:

$$p^*_{uuu} = p^*_u.p^*_{u/u}.p^*_{u/u/u},$$

$$p^*_{uud} = p^*_u.p^*_{u/u}.p^*_{d/u/u},$$

$$p^*_{udu} = p^*_u.p^*_{d/u}.p^*_{u/d/u},$$

$$p^*_{udd} = p^*_u.p^*_{d/u}.p^*_{d/d/u},$$

$$p^*_{duu} = p^*_d.p^*_{u/d}.p^*_{u/u/d},$$

$$p^*_{dud} = p^*_d.p^*_{u/d}.p^*_{d/u/d},$$

$$p^*_{ddu} = p^*_d.p^*_{d/d}.p^*_{u/d/d},$$

$$p^*_{ddd} = p^*_d.p^*_{d/d}.p^*_{d/d/d}.$$

Next we are going to value an option to invest which depends on three different stochastic processes by means of a three-dimensional binomial lattice.

4.2 Deploying the lattice

The time T until maturity is subdivided into n steps each of size $\Delta t = T/n$. In our case, after the first step the initial value I_0 moves to one of two possible values, $I_0 u_I$ or $I_0 d_I$, where $u_I = e^{\sigma_I \sqrt{\Delta t}}$ and $d_I = 1/u_I = e^{-\sigma_I \sqrt{\Delta t}}$. Starting from initial values (I_0, L_0, E_0) after the first step we can compute the values ($I_0 e^{\sigma_I \sqrt{\Delta t}}$, $L_0 e^{\sigma_L \sqrt{\Delta t}}$, $E_0 e^{\sigma_E \sqrt{\Delta t}}$) with probability p^*_{uuu}. Similarly we derive the remaining nodes that arise in the first step, for example ($I_0 e^{-\sigma_I \sqrt{\Delta t}}$, $L_0 e^{-\sigma_L \sqrt{\Delta t}}$, $E_0 e^{-\sigma_E \sqrt{\Delta t}}$) with probability p^*_{ddd}.

After i steps, with j_I, j_L and j_C upside moves, the values ($I_0 e^{\sigma_I \sqrt{\Delta t}(2j_I - i)}$, $L_0 e^{\sigma_L \sqrt{\Delta t}(2j_L - i)}$, $E_0 e^{\sigma_E \sqrt{\Delta t}(2j_E - i)}$) will be reached. It can easily be seen that the tree branches recombine; thus, the same value results from a rise followed by a drop or the other way round. At the final time T the possible combinations of values can be represented by means of a cube. At the earlier moment $T-\Delta t$ another less-sized cube describes the set of feasible values. There will be some probabilities of moving from each node to eight possible states of the cube at time T.

This lattice is used to assess the possibility to invest in enhancing the energy efficiency (thus saving input fuel and emission allowances) of a physical facility already in place (such as an operating coal-fired plant). Therefore, the saving opportunity is linked to the remaining life of the facility to be upgraded. We also consider that, once the decision to invest is made, it takes time for this enhancement to start working. The example below assumes that implementation takes a whole year.[19] The investment opportunity is assumed to be available from initial time until T when the plant is closed down.[20] However, given the time to build required, exercising the option to invest after time $T-1$ will never pay off.[21]. So at T -1 the value of the option at all the nodes is zero; at that time it is not possible to decide to invest:

$$W=0,$$

In a lattice with n time steps at the final time we will have $(n+1)^3$ nodes; in the moment immediately before, the number will be n^3 nodes.

At earlier times, i.e., for $t<T-1$, the option value at each node in the lattice is:

$$\begin{aligned} W \quad = \quad & \max(V(I_t,L_t,C_t), e^{-r\Delta t}(p^*_{uuu}W^{+++}+p^*_{uud}W^{++-}+p^*_{udu}W^{+-+}+p^*_{udd}W^{+--} \\ & +p^*_{duu}W^{-++}+p^*_{dud}W^{-+-}+p^*_{ddu}W^{--+}+p^*_{ddd}W^{---}). \end{aligned} \tag{20}$$

The NPV of investing immediately (i.e., exercising the option) is computed each time and it is compared with the second term, namely the value of the investment option kept alive. The maximum between them is finally chosen. W^{+++} denotes the value reached when moving from the current node to another one where the three variables have moved upward. The latter value has been already derived since the lattice is solved backwards. Note that the value of an investment at time $t<T-1$ must be computed between dates $t+1$ and T, i.e., one year after the investment decision until the facility's expiration at T. So in this case $\tau_1=t+1$ and $\tau_2=T$.

Proceeding backwards through the lattice we get an amount which shows the value of the option to invest, which cannot be negative.[22]

By changing the initial values $(I_0,L_{0,}C_0)$ we can derive those combinations for which the option to invest switches from worthy to worthless. These values

[19] There is a lapse since the decision to invest is made until the physical units that improve EE are received, trial tests are time-consuming, and also final adjustments.

[20] Therefore we deal with an American-type option with three sources of risk.

[21] We are assuming a fixed (deterministic) useful life of the physical asset.

[22] The three-dimensional lattice can require a lot of computer memory. It may be convenient to keep in the memory at a time only the two cubes we are working with at that time, namely those at times t and $t+\Delta t$.

provide the trigger prices for investing initially to be optimal. They also allow to draw the border between the "invest" region and the "wait" region.

5 RESULTS AND SENSITIVITY ANALYSIS

5.1 Results in the base case

For convenience we show again in Table 6 the parameter values adopted in the base case.

Table 6. Parameter values: base case.		
L_0	Current long-term price of coal	46.90
C_0	Spot price of emission allowance	17.8231
σ_I	Volatility of investment cost	0.10
σ_L	Volatility of coal price	0.2850
σ_C	Volatility of allowance price	0.5254
$\frac{K_L L_m}{K_L - \lambda_L}$	Long-term price of coal	70.13
ρ_{LC}	Correlation between coal and carbon	0.0525
$\alpha_I - \lambda_I$	Drift rate of investment cost	0
$\alpha_C - \lambda_C$	Drift rate of allowance price	0.056
$k_L + \lambda_L$	Reversion coefficient of coal price	0.62

In our computations we take 12 steps per year. The remaining life of the facility goes from 2 to 15 years. The number of steps is given by $12 * (T - 1)$. With 15 years this means 168 time steps. Therefore, the number of possible option values at time $T - 1$ when we start proceeding backwards is $4,826,809$; we assign them a value of zero. Of course, this will not be necessarily so at the $4,741,632$ nodes immediately before (at time $T - 13/12$).

We are going to make a first assessment with the initial values and assuming three possible investment costs I_0: 500, 750, and $1,000$. The value of the option to invest W consists of the value of investing immediately (NPV) and that of the option to wait; it is shown in Table 7. The option value clearly depends on the remaining useful life of the facility.

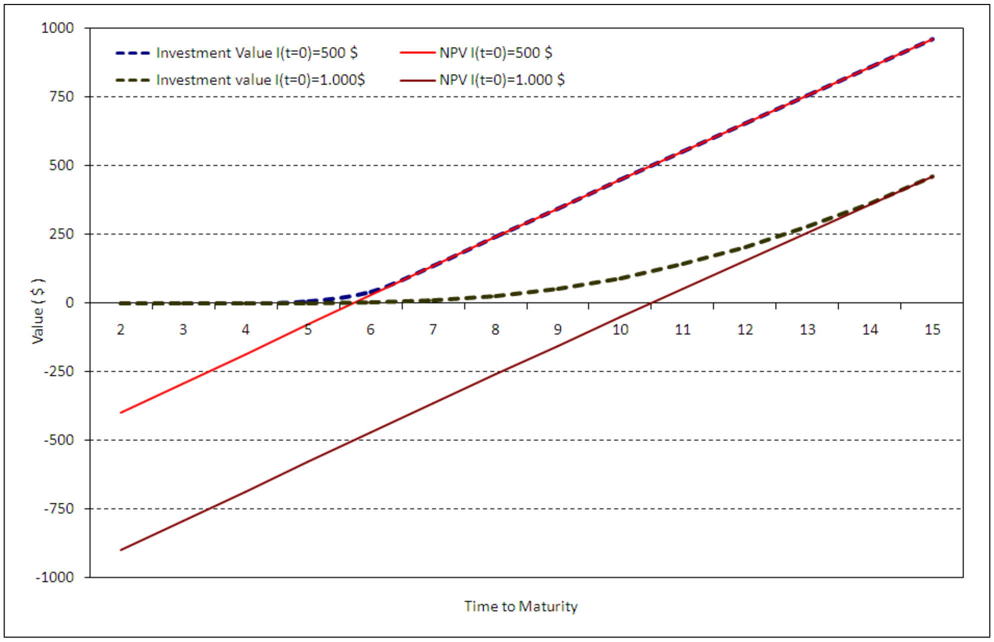

Figure 13: Value of the immediate investment and value of the option to invest.

Table 7. Value of the option to invest.

	$I_0 = 500$		$I_0 = 750$		$I_0 = 1,000$	
T	W	NPV	W	NPV	W	NPV
15	961.5	961.5	711.5	711.5	461.5	461.5
14	859.7	859.7	609.7	609.7	365.4	359.7
13	757.6	757.6	507.6	507.6	279.6	257.6
12	655.2	655.2	405.2	405.2	205.1	155.2
11	552.4	552.4	302.4	302.4	142.2	52.4
10	449.0	449.0	199.0	199.0	91.2	-51.0
9	345.1	345.1	117.2	95.1	52.5	-154.9
8	240.6	240.6	60.1	-9.4	25.6	-259.4
7	135.4	135.4	24.8	-114.6	9.6	-364.6
6	42.7	29.5	6.9	-220.5	2.3	-470.5
5	7.3	-77.0	0.9	-327.0	0.2	-577.0
4	0.3	-184.0	0.0	-434.0	0.0	-684.0
3	0.0	-291.0	0.0	-541.0	0.0	-791.0
2	0.0	-397.0	0.0	-647.0	0.0	-897.0

As shown in the first column ($I_0 = 500$), between year 6 and year 7 we switch from a situation in which $W > NPV$ to another in which $W = NPV$. Therefore, at some time in between the option to wait has become worthless. It will be optimal to invest when the remaining life at least equals that time (with $I_0 = 500$). And it will be optimal with just that time to maturity if the investment cost falls below $I_0 = 500$. For terms lower than or equal to 6 years and $I_0 = 500$, it is preferable to wait. And for terms longer than 7 years, it is optimal to invest immediately. The blue and red lines in Figure 13 show this result. The red line describes the NPV, i.e., the value of investing immediately. As such, it is negative when there are few years left to profit from the improvement in EE, while it becomes positive for longer operation periods. The blue line describes the value of the option to invest. Since it represents a right, not an obligation, its value cannot be negative. As can be seen, with few years left, the best decision is to wait, i.e., to keep the option alive (by not investing). For longer maturities, though, the investment can pay off, and waiting no longer makes sense (the two lines overlap each other).

The green and brown lines in Figure 13 above show a similar pattern for $I_0 = 1,000$. They contact each other somewhere between years 14 and 15. Now, a 50% subsidy of the initial investment costs which were only available at the outset would lead us to compare the option to invest later (green curve) with the NPV of an investment with a cost of 500 (red curve). The lines cross between years 5 and 6, thus making it easier to undertake the investment earlier in time, some six years before closure .[23]

5.2 Sensitivity analysis

Next we derive the threshold investment cost I^* (or optimal exercise price) in the base case and its sensitivity to changes in parameter values.

5.2.1 Sensitivity to changes in investment cost

We are going to analyze the impact of changes in the volatility σ_i and the drift rate $\alpha_I - \lambda_I$. The results appear in Table 8.

[23] We do not address the free-riding problem that can be triggered by this subsidy.

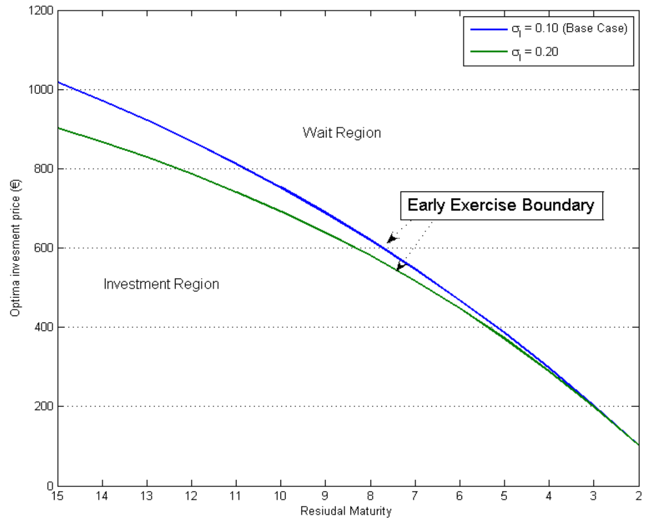

Figure 14: Threshold investment cost for different cost volatilities as a function of the facility's remaining life.

Table 8. Sensitivity to investment cost.				
	Change in σ_I		Change in $\alpha_I - \lambda_I$	
T	$\boldsymbol{\sigma_I = 0.10}$	$\sigma_I = 0.20$	$\alpha_I - \lambda_I = 0.025$	$\alpha_I - \lambda_I = -0.025$
15	1,019.2	902.0	1,069.1	958.7
14	972.4	866.7	1,015.8	920.1
13	922.5	928.5	959.8	878.0
12	869.4	787.0	900.8	832.3
11	812.0	741.8	838.8	782.5
10	752.5	692.6	773.4	728.4
9	688.3	639.0	704.5	669.7
8	619.7	580.5	631.8	606.0
7	546.5	516.7	555.1	537.0
6	468.4	447.1	474.0	462.3
5	385.0	371.0	388.4	381.6
4	296.2	288.1	297.9	294.7
3	202.0	198.3	202.6	201.5
2	102.5	101.6	102.6	102.6

As expected, a higher cost volatility σ_I raises the strain in the form of a lower level I^* for the investment cost. With 15 years to maturity, an 11.4% fall in cost

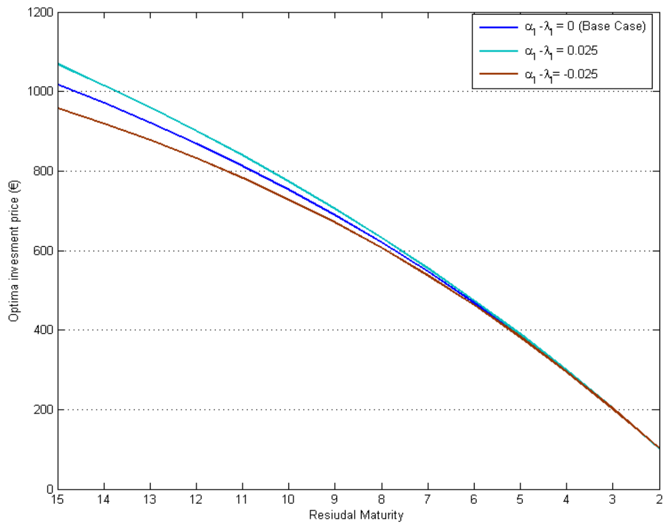

Figure 15: Threshold investment cost for different cost drift rates as a function of the facility's remaining life.

($I^* = 1,019.2$) is required with respect to the base case ($\sigma_I = 0.10$). Figure 14 shows that the threshold cost is lower for higher volatilities. A more uncertain environment regarding costs leads managers to delay investments unless their costs fall relatively to the former situation. Needless to say, as the facility gets closer to its end, the threshold cost falls consistently.

A growth rate $\alpha_I - \lambda_I = 0.025$ in the risk-neutral world makes investment easier by pushing the level I^* upward. Conversely, a rate $\alpha_I - \lambda_I = -0.025$ compounds investment at the outset, since its cost is expected to decrease in the future.[24] Figure 15 shows this effect: if the project costs are expected to increase significantly in the future, it is relatively better to invest sooner (rather than later), so the managers are less demanding in terms of I^*. Therefore, the curve shifts upwards.

5.2.2 Sensitivity to changes in the emission allowance price

Let us consider the case of a change in the initial allowance price and allowance volatility. See Table 9.

[24] For those interested in the effectiveness of subsidies to investment cost as a measure to entice firms into EE, see Jaffe and Stavins (1994) and Hassett and Metcalf (1995).

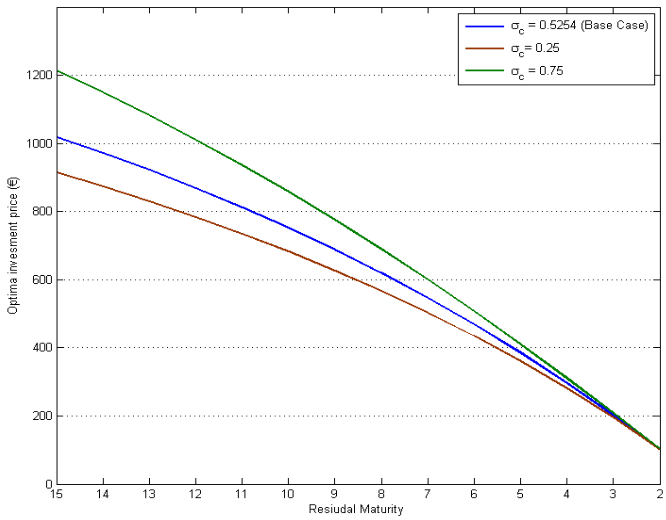

Figure 16: Threshold investment cost for different allowance volatilities as a function of the facility's remaining life.

Table 9. Sensitivity to emission allowance.					
		Change in σ_C		Change in $\alpha_C - \lambda_C$	Change in C_0
T	$\sigma_C = 0.25$	$\boldsymbol{\sigma_C} = \mathbf{0.5254}$	$\sigma_C = 0.75$	$\alpha_C - \lambda_C = 0.10$	$C_0 = 30.00$
15	1,213.8	1,019.2	915.2	1,093.5	1,200.7
14	1,149.5	972.4	873.9	1,042.0	1,149.0
13	1,081.8	922.5	830.2	987.2	1,093.8
12	1,010.5	869.4	783.9	928.9	1,035.0
11	935.7	812.0	734.7	866.9	971.5
10	857.2	752.5	682.4	800.9	903.6
9	775.1	688.3	626.5	730.7	830.6
8	689.4	619.7	566.8	656.1	752.2
7	599.9	546.5	503.0	576.8	667.7
6	506.8	468.4	434.4	492.5	576.5
5	410.1	385.0	360.6	403.2	478.0
4	310.1	296.2	280.9	308.8	371.5
3	207.4	202.0	194.6	209.4	256.6
2	103.1	102.5	101.2	105.6	132.5

A low allowance price volatility raises significantly the threshold cost below which we would be ready to invest. As shown in Figure 16, a higher allowance volatility feeds cautiousness in that managers require lower investment costs in

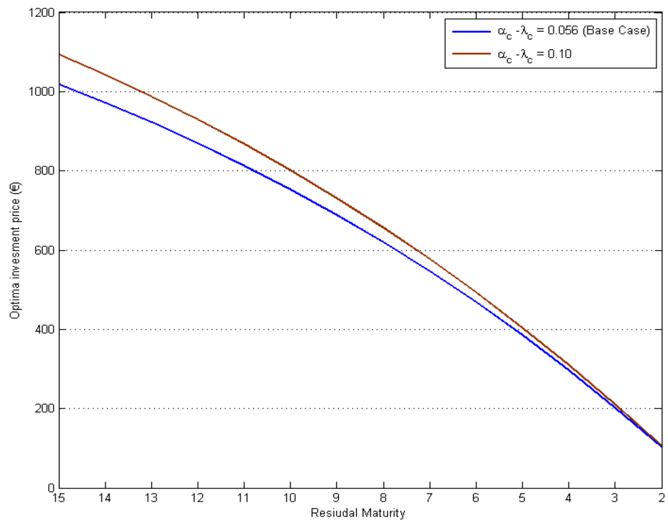

Figure 17: Threshold investment cost for different allowance drift rates as a function of the facility's remaining life.

order to undertake the project.

An increase in the slope, i.e., a higher allowance price expected in the future, eases investments to enhance energy efficiency. Anticipation of higher allowance prices in the future means higher savings to be reaped from improving EE. Therefore, as Figure 17 suggests, investment can be justified for higher costs.

Figure 18 shows that allowance prices have an acute impact on decisions to invest in EE. According to the last column in Table 9, if initial carbon prices are higher (30.00 instead of 17.82), managers will be persuaded to pay higher investment costs.

5.2.3 Sensitivity to changes in coal price

We analyze changes in volatility and the long-term price. Volatility has a rather limited impact. This is due to the strong effect of the mean-reversion coefficient. Table 10 shows these results.

Table 10. Sensitivity to changes in σ_L.

T	$\sigma_L = 0.25$	$\boldsymbol{\sigma_L} = \mathbf{0.2850}$	$\sigma_L = 0.45$
15	1,020.6	1,019.2	1017.0
10	753.4	752.5	751.0
5	385.5	385.0	384.4

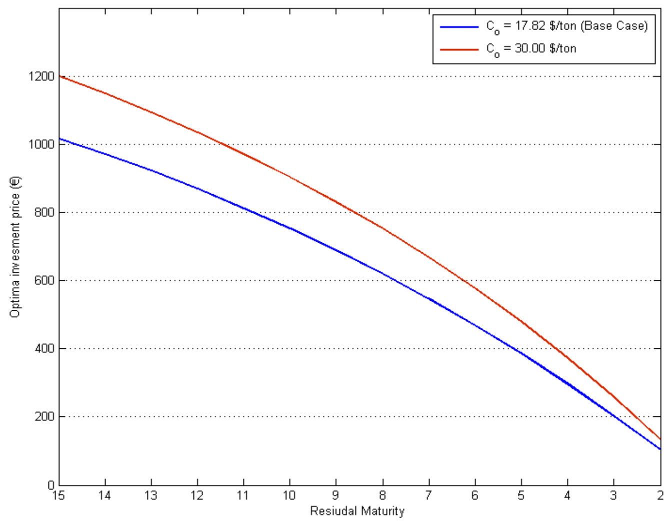

Figure 18: Threshold investment cost for different allowance prices as a function of the facility's remaining life.

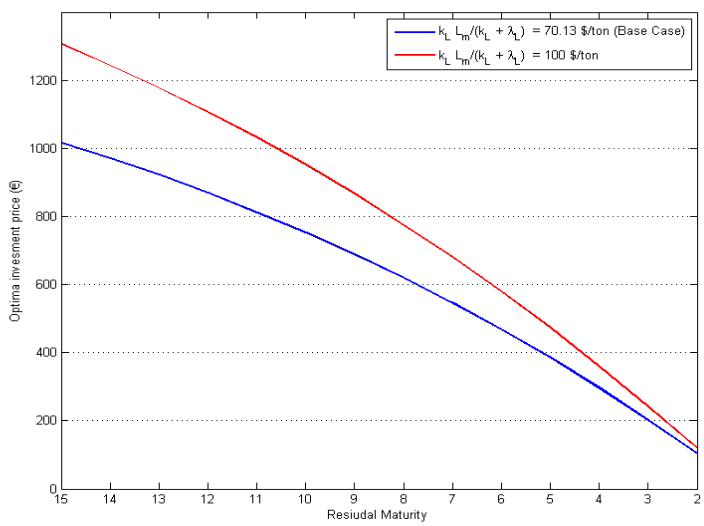

Figure 19: Threshold investment cost for different long-term coal prices as a function of the facility's remaining life.

Instead, the long-term price of coal has a large impact. See Table 11. Again, if coal prices are higher, investments to enhance EE will be more easily justified from a financial point of view. As shown in Figure 19, higher savings in energy bills allow the trigger cost I^* to move upwards.

Table 11. Sensitivity to changes in L_m.

T	$\frac{K_L L_m}{K_L-\lambda_L} = \mathbf{70.13}$	$\frac{K_L L_m}{K_L-\lambda_L} = 100.00$
15	1,019.2	1,306.7
14	972.4	1,244.2
13	922.5	1,177,7
12	869.4	1,107.0
11	812.0	1,031.9
10	752.5	952.1
9	688.3	867.3
8	619.7	773.3
7	546.5	681.3
6	468.4	580.1
5	385.0	472.7
4	296.2	359.7
3	202.2	241.2
2	102.5	119.7

6 CONCLUDING REMARKS

Energy efficiency (EE) investments can help reduce both energy and GHG emission bills. According to the IEA (2009), end-use and power plants efficiency (including new appliances, more efficient gas and coal plants, switching from coal to gas and early retirements) can deliver globally 8 GtCO_2 of abatement by 2030, the same amount than nuclear, renewable and CCS technologies together. For those savings to become a reality the IEA estimates that additional investment of 7,500 billion ($2008) will be needed until 2030.

Although the potential of EE seems huge, it is not being fully undertaken. Some will argue that this so-called energy efficiency paradox is not such. Given rational consumers and efficient markets, investments observed are economically optimal; any deviation from optimality would be explained by hidden costs. Others, however, would indicate that energy markets are subject to failures and barriers that explain this gap.

In any case, there is one element that can surely explain a part of the story behind the "efficiency gap"; namely the (lack of) consideration of uncertainty when valuing potential returns on these projects. If uncertainties are not addressed conveniently decision makers can choose for inaction despite investment being profitable, or choose for action despite being unprofitable. In fact, the returns on EE investments depend heavily on variables that by their very nature are not deterministic, e.g., regulatory framework, energy prices, or emission permit restrictions.

In this paper we consider uncertain costs and revenues from projects that enhance EE; our aim is to determine the optimal time to invest. Investment is valued like a (real) option that is only exercised at the optimal time, and is irreversible. There are three sources of uncertainty: the long-term dynamics of the commodity (coal) price, the emission allowance price, and the overall investment cost. A stochastic model is calibrated for these three sources. We assume that the commodity price follows a mean-reverting stochastic process. Regarding the allowance price and the investment cost we adopt a geometric Brownian motion. Parameter values for these price processes have been calibrated from samples of futures prices of coal (NYMEX) and EU emission allowances (ECX). Then we can compute the value of a stochastic annuity from fuel saved and allowances spared. After subtracting the investment cost the Net Present Value (NPV) of the project results.

In particular, we have considered an operating physical facility already in place with a remaining useful life that ranges from 2 to 15 years. The investment to improve EE takes a whole year to be operative. The numerical estimates of the parameters are then used in a three-dimensional binomial lattice to assess the value of the option to invest. We note that our procedure precludes the possibility of negative probabilities. Maximizing the option value involves determining the optimal exercise time. Thus we compute the trigger investment cost, i.e., the threshold level below which immediate investment would be optimal. To our knowledge, a three-dimensional lattice allowing for mean-reverting processes has not been previously solved and used in any application.

Our results show the NPV of an immediate investment along with the value of the option to invest for different investment costs (500, 750, 1000). When the value of waiting is zero we would invest immediately. In the base case (I_0 = 500) investment will be optimal for remaining lives beyond some six years. For terms lower than or equal to six years it is preferable not to invest even if the NPV is positive. This finding can help understand the "efficiency gap" and the different perspectives sometimes adopted by engineers and economists when valuing projects. Moreover, as investment cost increases, exercising the option requires longer periods of useful life. Thus, doubling the cost (I_0 =1000) makes investment optimal only when the remaining life is 15 years, even though the NPV of the investment would positive for 10 years.

We have assessed several policy measures in terms of their influence on the optimal time to invest in EE improvements. Indeed, regulators can play an important role in bringing forward these investments: (a) given the external positive effects resulting from EE investments (climate change, health benefits, security of supply), a public subsidy can be justified; (b) uncertainties must be reduced where possible (e.g. regarding the EU ETS, the post-Kyoto scenario, etc.); (c) policy makers can raise carbon prices by reducing the supply of allowances. If these measures are taken in a transparent manner, within a long-term framework, so much the better.

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