Wavelet spectral analysis of unevenly spaced paleoclimate time series using the Morlet Weighed Wavelet Z-Transform

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ABSTRACT

Paleoclimate time series are generally short and noisy, do not have many elements, may contain periodic and quasi-periodic events or transient signals, and have a particular drawback: it is not always possible to control the sampling intervals, therefore are unevenly spaced [8, 12, 14]. The most common way for overcoming this inconvenient is to interpolate in time the original unevenly spaced time series in order to obtain equidistance. However, interpolation tends to over-estimate the low-frequencies, under-estimate the high-frequencies, and could also introduce spurious spectral peaks. For all these reasons, many studies suggest that interpolation should be avoided [8, 12, 13]. A more effective way to tackle this problem in the time-frequency domain would be to use the continuous Morlet Weighted Wavelet Z-Transform (MWWZ) [2], which is able to handle directly unevenly spaced time series. In this work, we present a preliminary statistical-computational implementation of the MWWZ to estimate the Wavelet Power Spectrum, showing its potential use in paleoclimate research.

1 Motivation:

1.1 Main characteristics of paleoclimate time series (faced in this work):

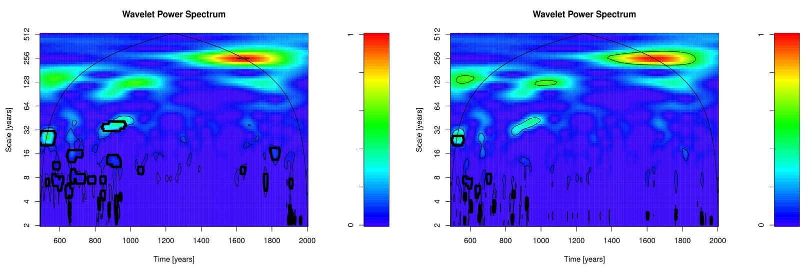
- are **noisy**
- are **not stationary**
- contain information in several temporal scales (**multiscale**)
- may contain **period**, **quasi events** or **transient signals**

⇒ and are usually **unevenly spaced**

1.2 Solutions:

- **INTERPOLATION**, but interpolation alters the estimated spectrum [8, 12, 13]:

Fig. 1 Effects of interpolation (linear vs. nearest-neighbor, left and right, respectively) on the Average Wavelet Power Spectrum WPS. WPS has been estimated following the methodology of [5]. Thin and tick lines, are the pointwise and areawise test, respectively. The unevenly spaced time series (δ^{18}O) was obtained from [6].



- **SPECIAL (FOURIER) SPECTRAL ANALYSIS** techniques to handle unevenly data, such as: **Lomb-Scargle Periodogram** via **REDFIT** [12], **CLEAN** [4], **the Nava technique** [10], etc., but these methods are designed for the analysis of stationary data and it is limited to look through the frequency domain.

- **AN ADEQUATE SOLUTION IS** the **continuous Morlet Weighed Wavelet Z-Transform (MWWZ)**

Currently, there is an implementation of **MWWZ** in the literature that can be used with paleclimate time series [15]. However, neither the details of this computational implementation nor the supporting software are currently available. Thus, it is necessary a statistical-computational implementation freely available to the paleo/climate community.

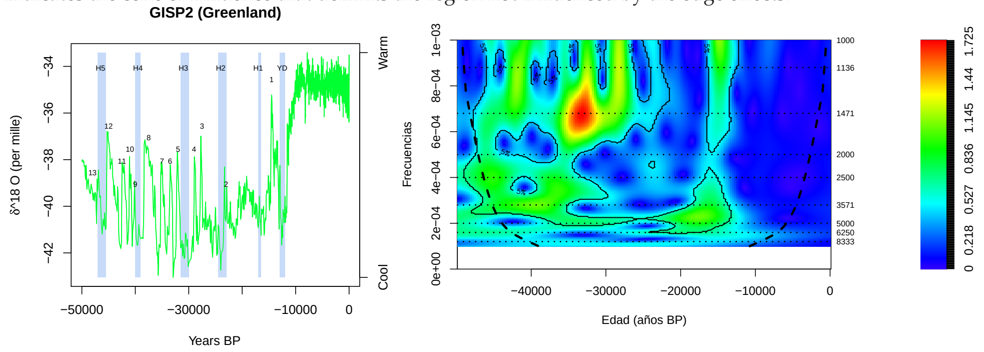
2 Methodology: WPS estimation

1. *Input* ⇐ unevenly spaced paleoclimate time series under study $x(t_i)$ (the mean and the linear trend have been removed)
2. Define some key parameters:
   - c = 0.01266515 (the decay constant) [2, 15]
   - the maximum/minimum frequencies
   - deltaf (the distance between frequencies)
   - τ compute the persistence coefficient to $x(t_i)$, which is performed using the TAUEST program [7])
3. Compute the Weighted Wavelet Amplitude WWA [2] to $x(t_i)$. We have used a program in Fortran to compute the WWA obtained from the American Association of Variable Star Observers (AAVSO) `http://www.aavso.org/software-directory`.
4. Monte Carlo simulation loop: For $i = 1$ to Nsims (at least 2000 simulations)
   a) Generate AR1(t_i) unevenly time series (after Eq. 2 in Ref. [12]) using the sampling times of $x(t_i)$ and the estimated τ.
   b) Compute the weighted wavelet amplitude WWA to each synthetic AR1(t_i) time series: WWA[AR1(t_i)]
5. Compute the 95th percentile of the ensemble: WWA[AR1(t_i), i=1, Nsims]
6. To discern if a spectral point of WWA[$x(t_i)$] is statistically significantly different from zero (to the 95% confidence level), it is compared with the 95th perc. of the ensemble.
7. *Output* ⇒ The Wavelet Power Spectrum with its significant spectral points are shown, for example, in Figure 2.

The statistical-computational algorithm (except the WMA) and the heat maps have been programmed in R [11].

3 Preliminary results

Fig. 2 (Left side) GISP2 δ^{18}O record (unevenly spaced). The gray vertical boxes are the **Heinrich events H1-H5** and the **Younger Dryas event (YD)** and the numbers from 1 to 13 are the **Dansgaard-Oeschger** events. (Right side) Wavelet Power Spectrum (not smoothed) via MWWZ of the GISP2 δ^{18}O record. Contour lines indicate wavelet amplitudes that are significantly (95% CI)) project above a red noise background. The back dashed line indicates the cone of influence that delimits the region not influenced by the edge effects.



Remarkable preliminary results:

1. Our statistical-computational program is able to detect the prominent **1470 yr spectral peak** centered around -35 to -31 Kyr time interval (Fig. 2 right side), a well-known spectral signature of the GISP2 ice core record [3].
2. The wavelet structures detected (Fig. 2 right side) are relatively close to the results obtained by Witt & Schumman [15], who used the MWWZ to analyze the GISP2 δ^{18}O record (-50,000 to 0 BP time interval).
3. The simple statistical significance test used in this work is quite flexible considering spectral points/areas as a significants. Therefore, the results obtained should be considered with caution.

4 Conclusions and Future Work

The statistical-computational implementation to estimate the Wavelet Power Spectrum presented in this work, provides a useful tool to analyze directly unevenly spaced paleoclimate time series in the frequency-time domain. However, there are some improvements that must be taken into account for future works:

- Include an averaging (smoothing) in the time and frequency domain
- Include the Andronov's improvements [1] (additional weighting factors)
- Improve the statistical significance test
- Considerer the timescale errors (uncertainty in the dating of the paleoclimate records and errors associated with the construction of age models [8, 9]) in the estimation of the Wavelet Power Spectrum.
- Extend the MWWZ method to the bi-variate case: wavelet coherence and phase coherence (**something that has so far not been carried out**).

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